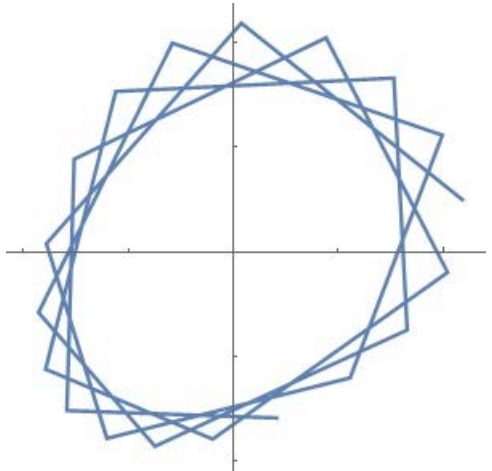
Cellular automata

theory

Notes for CAS

Feijs, Delbressine



Cellular automata:

- Model of interacting agents in fixed grid
- Very simple rules, yet complex behaviour
- Fundamental studies about computation and complexity
- Also useful for simulations in physics and social computation

Good overview: http://slideplayer.com/slide/5674720/

Mathworld: A cellular automaton is a collection of "colored" cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells.

The rules are then applied iteratively for as many time steps as desired. von Neumann was one of the first people to consider such a model, and incorporated a cellular model into his "universal constructor." Cellular automata were studied in the early 1950s as a possible model for biological systems (Wolfram 2002, p. 48).

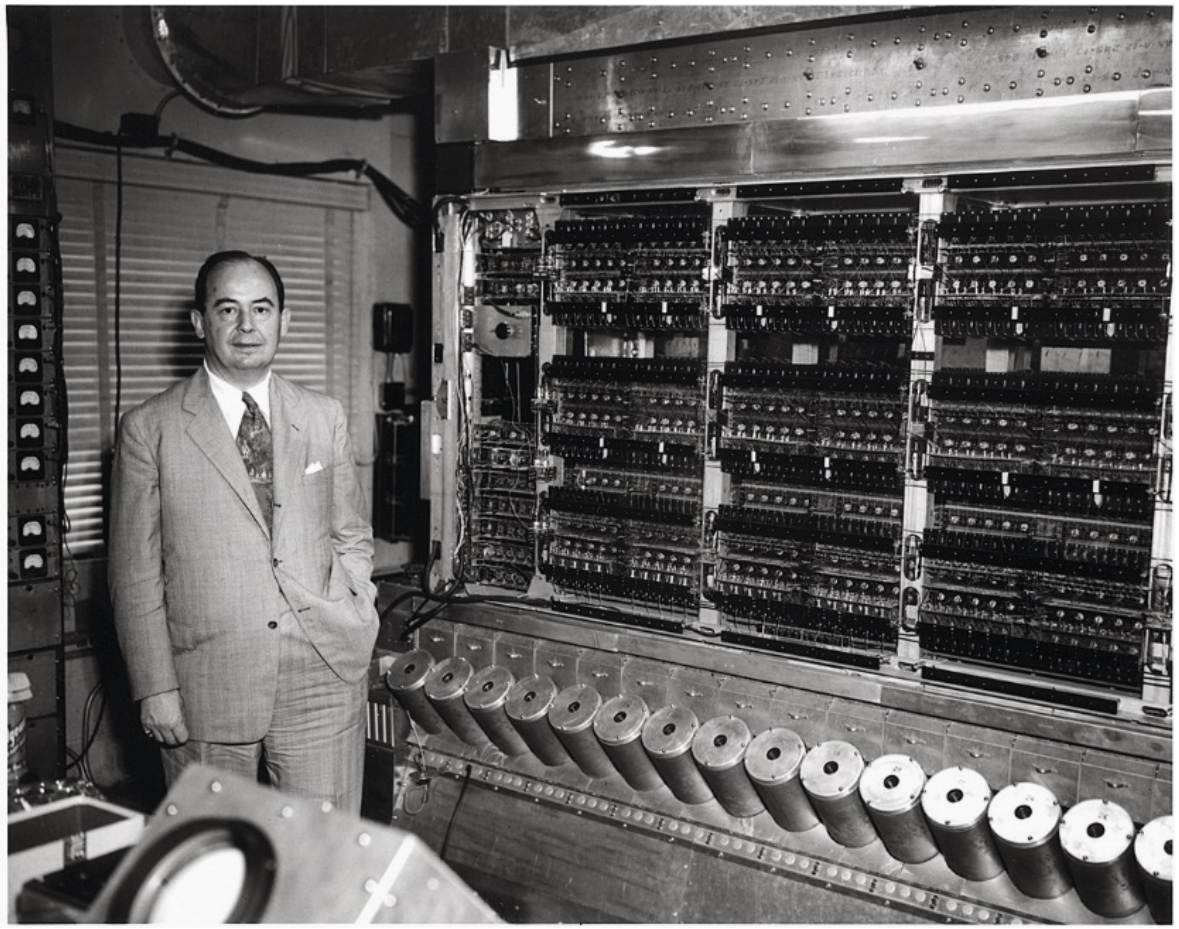
Comprehensive studies of cellular automata have been performed by S. Wolfram starting in the 1980s, and Wolfram's fundamental research in the field culminated in the publication of his book *A New Kind of Science* (Wolfram 2002) in which Wolfram presents a gigantic collection of results concerning automata, among which are a number of groundbreaking new discoveries.

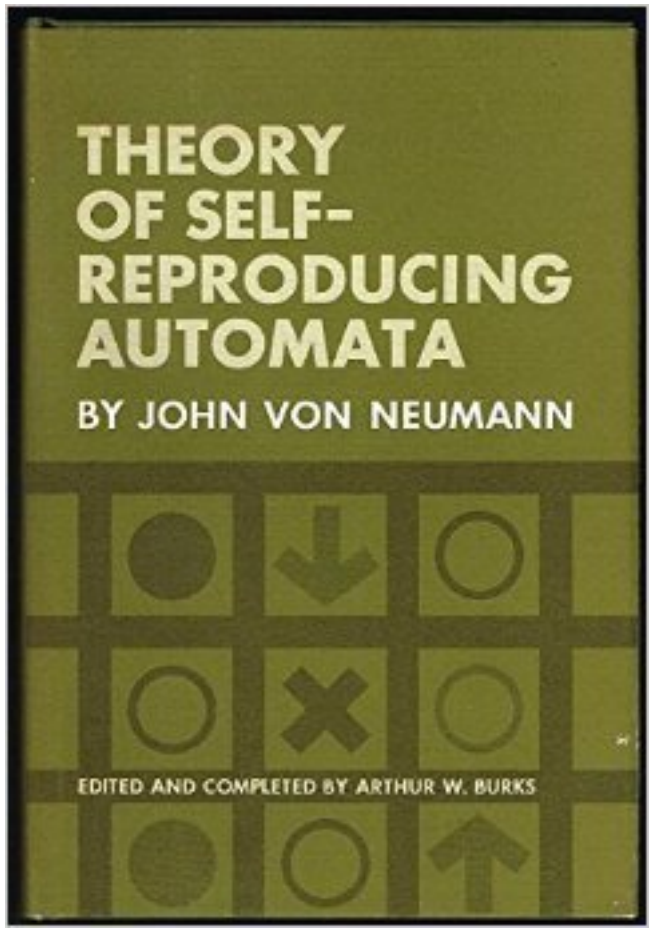
Cellular automata:

- every cell (agent) has finite-state machine behaviour
- neighbour states + own state determine new state
- all cells are updated simultaneously
- the rules are the same for all cells

History:

- John Von Neumann (1948, self-replication)
- John Conway (1970, universality)
- Steven Wolfram (2002, complexity)

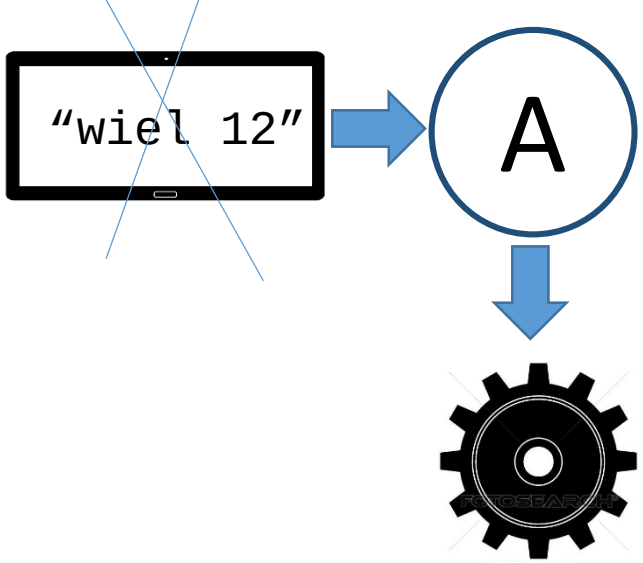


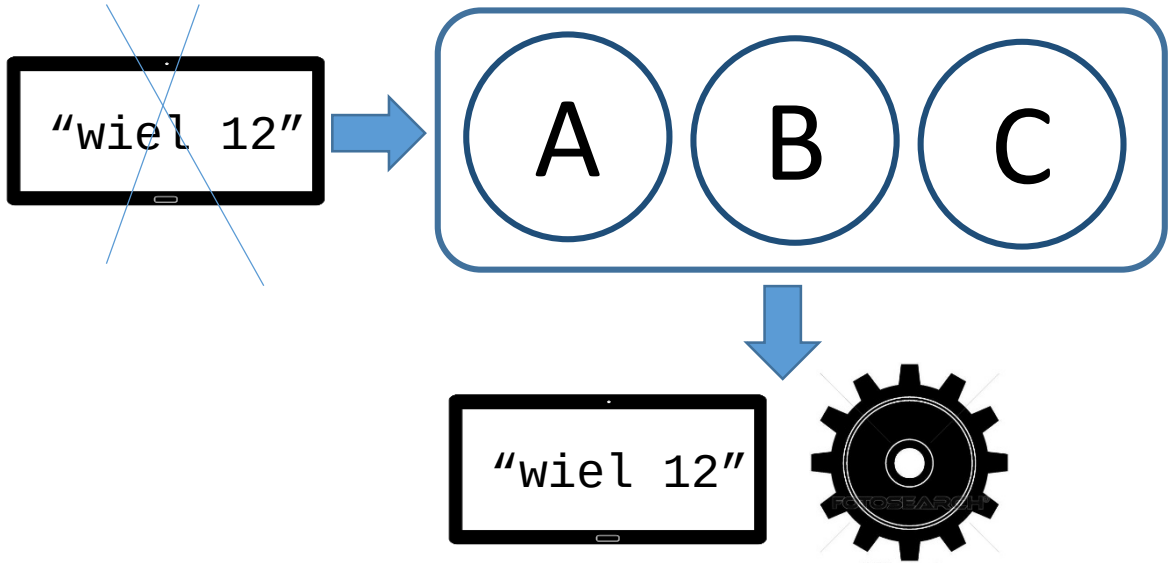

THEORY
OF SELF-
REPRODUCING
AUTOMATA
BY JOHN VON NEUMANN
EDITED AND COMPLETED BY ARTHUR W. BURKS

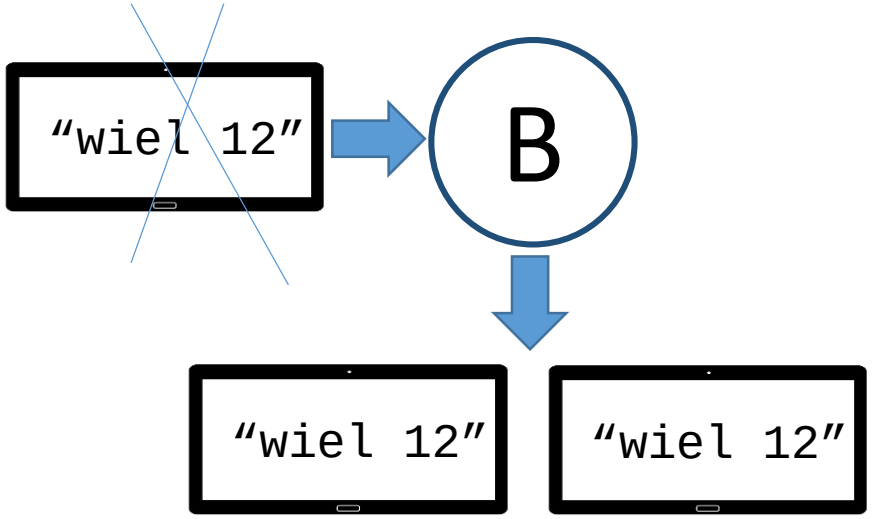
The general constructive automaton A produces only X when a complete description of X is furnished it, and on any reasonable view of what constitutes complexity, this description of X is as complex as X itself. The general copying automaton B produces two copies of $\phi(X)$, but the juxtaposition of two copies of the same thing is in no sense of higher order than the thing itself. Furthermore, the extra unit B is required for this copying.

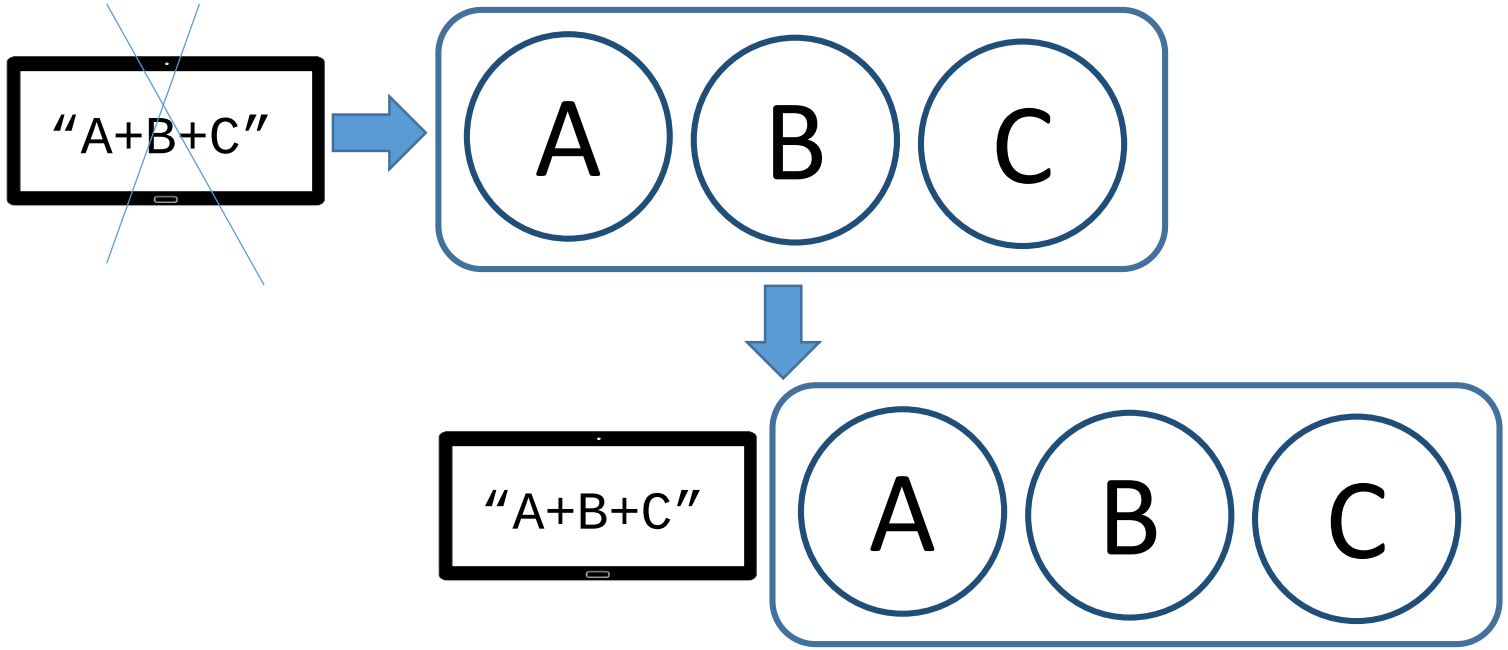
Now we can do the following thing. We can add a certain amount of control equipment C to the automaton $A + B$. The automaton C dominates both A and B, actuating them alternately according to the following pattern. The control C will first cause B to make two copies of $\phi(X)$. The control C will next cause A to construct X at the price of destroying one copy of $\phi(X)$. Finally, the control C will tie X and the remaining copy of $\phi(X)$ together and cut them loose from the complex $(A + B + C)$. At the end the entity $X + \phi(X)$ has been produced.

archive.org/details/theoryofselfrepr00vonn_0

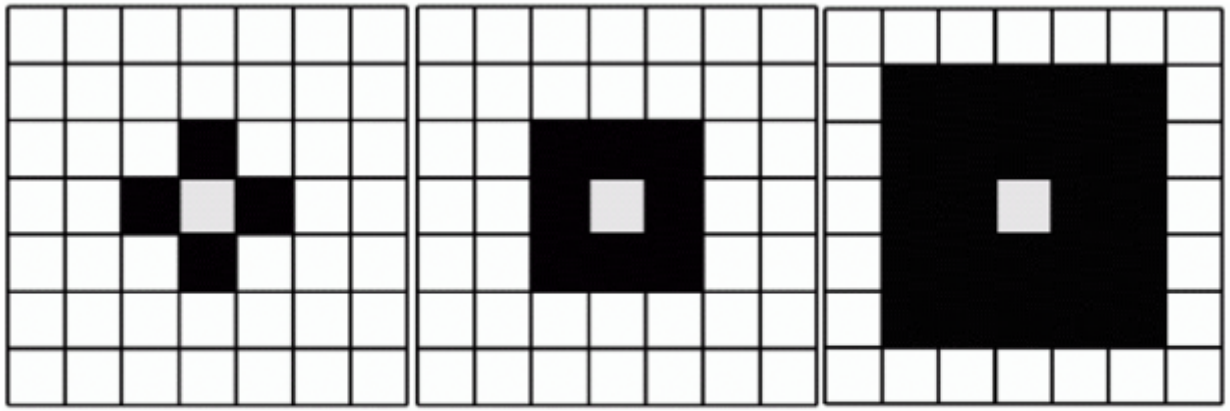

“wiel 12”
A


“wiel 12”
A
B
C
“wiel 12”


“wiel 12”
B
“wiel 12”
“wiel 12”


“A+B+C”
A
B
C
“A+B+C”
A
B
C

https://www.openabm.org/book/export/html/1949



von Neumann Neighborhood | Moore Neighborhood | Extended Moore Neighborhood

An Implementation of von Neumann's Self-Reproducing Machine

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Keywords
cellular automata, self-reproduction, universal constructor

Abstract This article describes in detail an implementation of John von Neumann's *self-reproducing machine*. Self-reproduction is achieved as a special case of construction by a *universal constructor*. The theoretical proof of the existence of such machines was given by John von Neumann in the early 1950s [6], but was first implemented in 1994, by the author in collaboration with R. Nobili. Our implementation relies on an extension of the state-transition rule of von Neumann's original cellular automaton. This extension was introduced to simplify the design of the constructor. The main operations in our constructor can be mapped into operations of von Neumann's machine.

a **ground** state **U** (48, 48, 48)

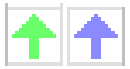
the **transition** or **sensitised** states (in 8 substates)

- **S** (newly sensitised) (255, 0, 0)
- $\mathbf{S_0}$ - (sensitised, having received no input for one cycle) (255, 125, 0)
- $\mathbf{S_{00}}$ - (sensitised, having received no input for two cycles) (255, 175, 50)
- $\mathbf{S_{000}}$ - (sensitised, having received no input for three cycles) (251, 255, 0)
- $\mathbf{S_{01}}$ - (sensitised, having received no input for one cycle and then an input for one cycle) (255, 200, 75)
- $\mathbf{S_1}$ - (sensitised, having received an input for one cycle) (255, 150, 25)
- $\mathbf{S_{10}}$ - (sensitised, having received an input for one cycle and then no input for one cycle) (255, 255, 100)
- $\mathbf{S_{11}}$ - (sensitised, having received input for two cycles) (255, 250, 125)

the **confluent** states (in 4 states of excitation)

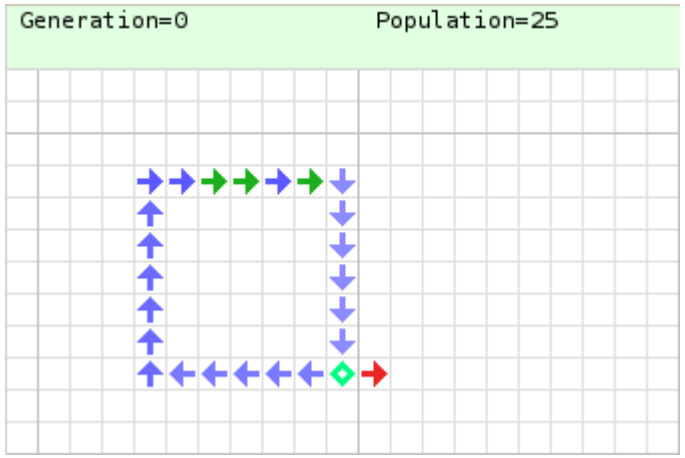
- $\mathbf{C_{00}}$ - quiescent (and will also be quiescent next cycle) (0, 255, 128)
- $\mathbf{C_{01}}$ - next-excited (now quiescent, but will be excited next cycle) (33, 215, 215)
- $\mathbf{C_{10}}$ - excited (but will be quiescent next cycle) (255, 255, 128)
- $\mathbf{C_{11}}$ - excited next-excited (currently excited and will be excited next cycle) (255, 128, 64)

the **ordinary transmission** states (in 4 directions, excited or quiescent, making 8 states)

- North-directed (excited and quiescent) 
- South-directed (excited and quiescent)
- West-directed (excited and quiescent)
- East-directed (excited and quiescent)

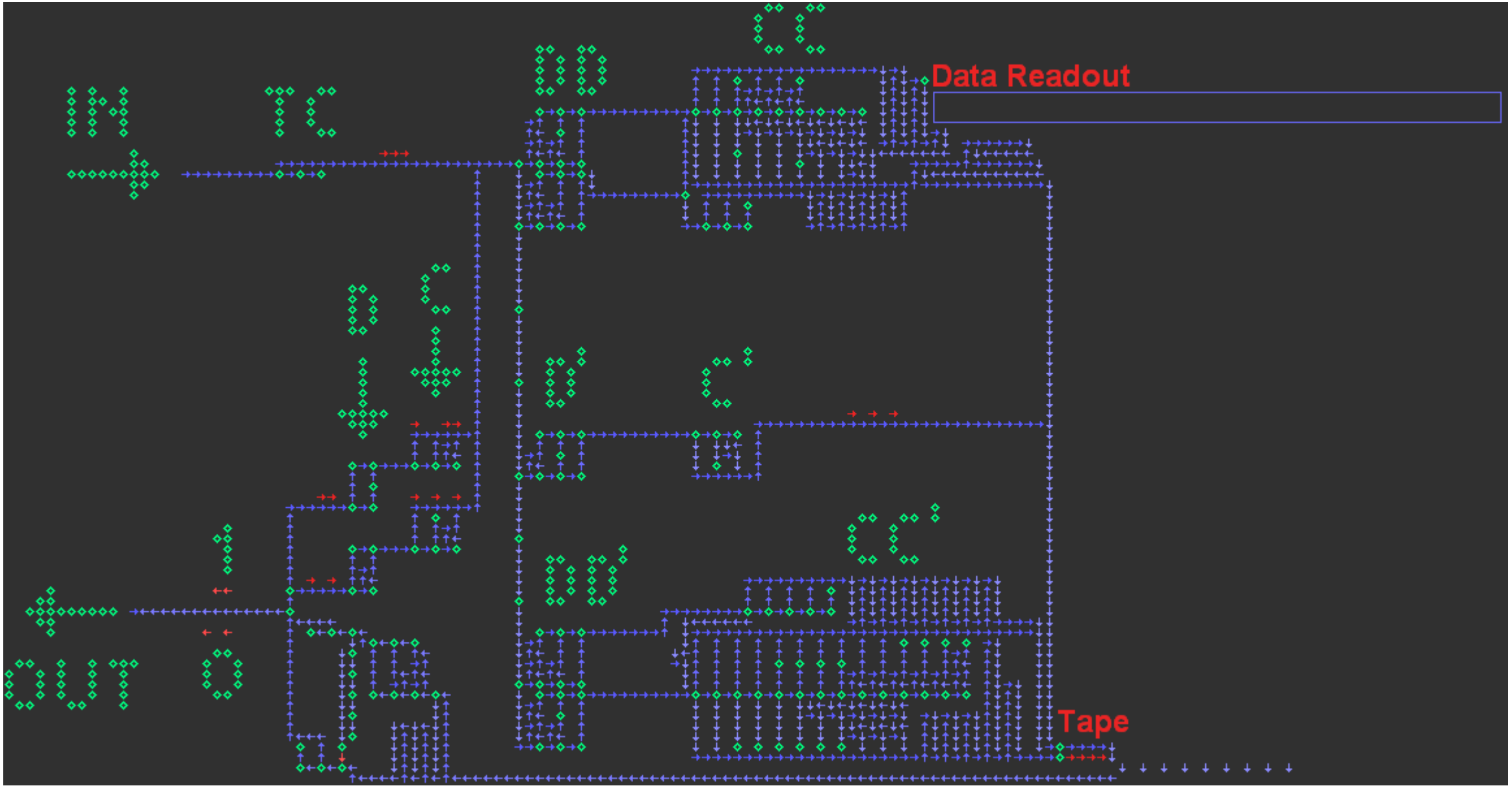
the **special transmission** states (in 4 directions, excited or quiescent, making 8 states)

- North-directed (excited and quiescent) (191, 73, 255) (255, 56, 56)
- South-directed (excited and quiescent) (203, 106, 255) (255, 89, 89)
- West-directed (excited and quiescent) (197, 89, 255) (255, 73, 73)
- East-directed (excited and quiescent) (185, 56, 255) (235, 36, 36)



A simple configuration in von Neumann's cellular automaton. A binary signal is passed repeatedly around the blue wire loop, using excited and quiescent *ordinary transmission states*. A confluent cell duplicates the signal onto a length of red wire consisting of *special transmission states*. The signal passes down this wire and constructs a new cell at the end. This particular signal (1011) codes for an east-directed special transmission state, thus extending the red wire by one cell each time. During construction, the new cell passes through several sensitised states, directed by the binary sequence

Source:
https://en.wikipedia.org/wiki/Von_Neumann_cellular_automaton



Source: https://en.wikipedia.org/wiki/Von_Neumann_cellular_automaton

QuineByPhiLho | Processing 2.2.1

File Edit Sketch Tools Help

Java

QuineByPhiLho

```
char q = 34, c = 44, n = 10;
String[] a =
{
"char q = 34, c = 44, n = 10;",
"String[] a =",
"{",
"",
"};",
"String B() { String s = new String(); for (int i = 0; i < a.length; i++) { s += q + a[i] + q + c + n; } return s; }",
"void D() { for (int i = 0; i < a.length; i++) println(a[i]); }",
"void setup() { a[3] = B(); D(); exit(); }",

};
String B() { String s = new String(); for (int i = 0; i < a.length; i++) { s += q + a[i] + q + c + n; } return s; }
void D() { for (int i = 0; i < a.length; i++) println(a[i]); }
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};
String B() { String s = new String(); for (int i = 0; i < a.length; i++) { s += q + a[i] + q + c + n; } return s; }
void D() { for (int i = 0; i < a.length; i++) println(a[i]); }
void setup() { a[3] = B(); D(); exit(); }
```

17

https://processing.org/discourse/beta/num 1243254687.html

Conway’s game of life

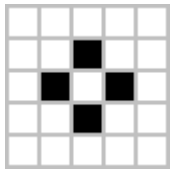
- Any live cell with fewer than two live neighbours dies, as if caused by underpopulation.
- Any live cell with two or three live neighbours lives on to the next generation.
- Any live cell with more than three live neighbours dies, as if by overpopulation.
- Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction.

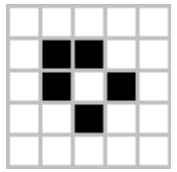
(https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life)

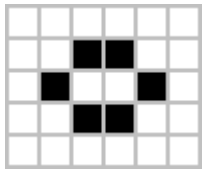
DIGITAL EQUIPMENT CORPORATIO

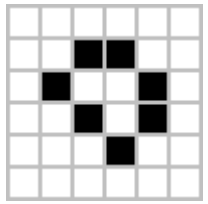
Static

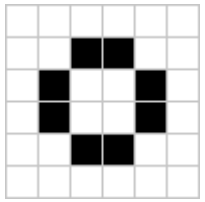
block



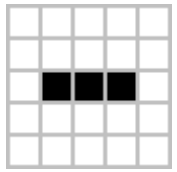


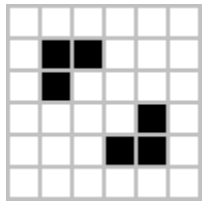

beehive

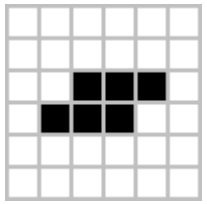


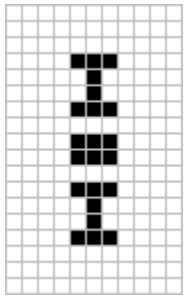

pond

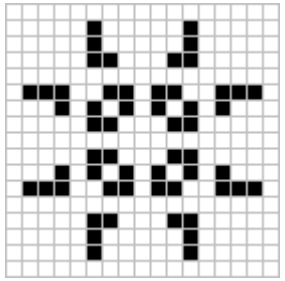
Pulsing


blinker

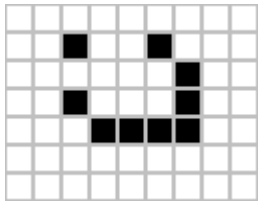


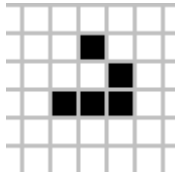


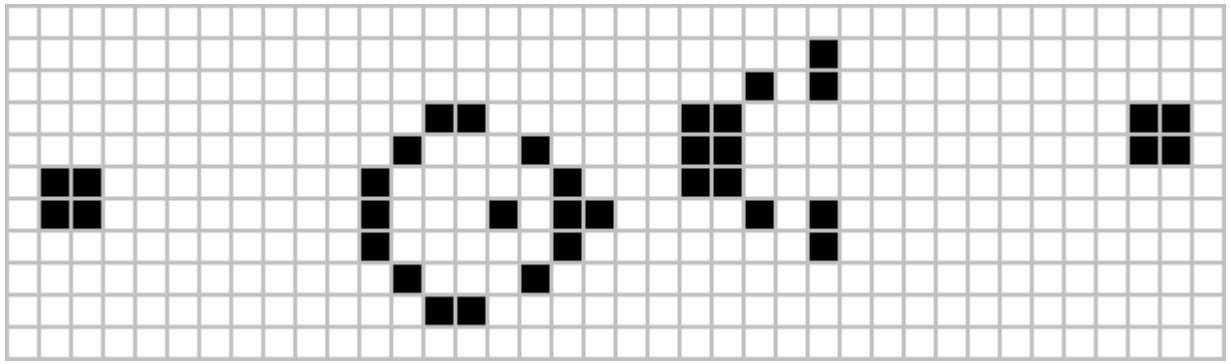




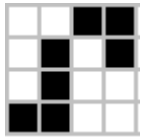
Crawling


lightweight spaceship (c/2)

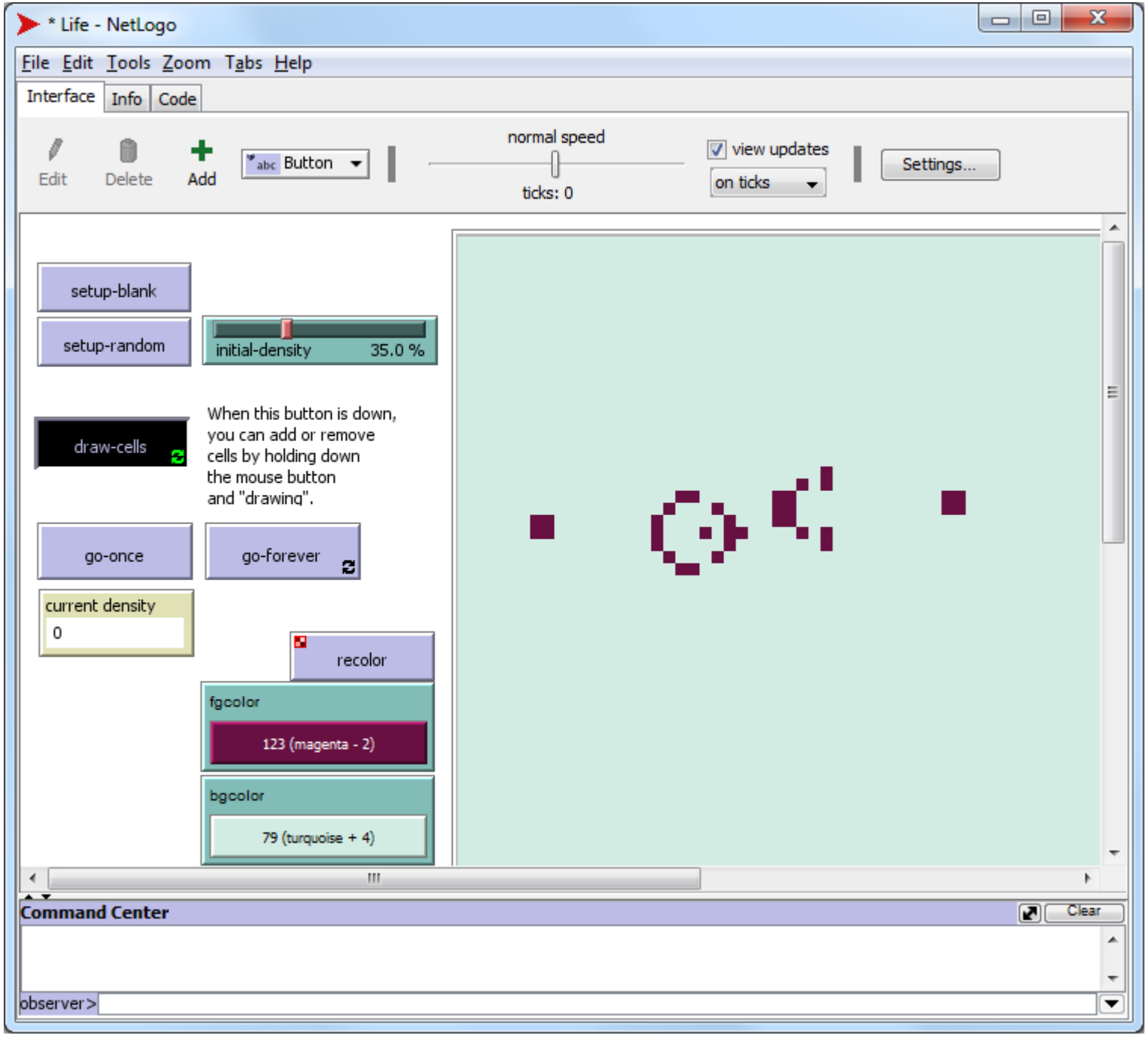

Glider (c/4)

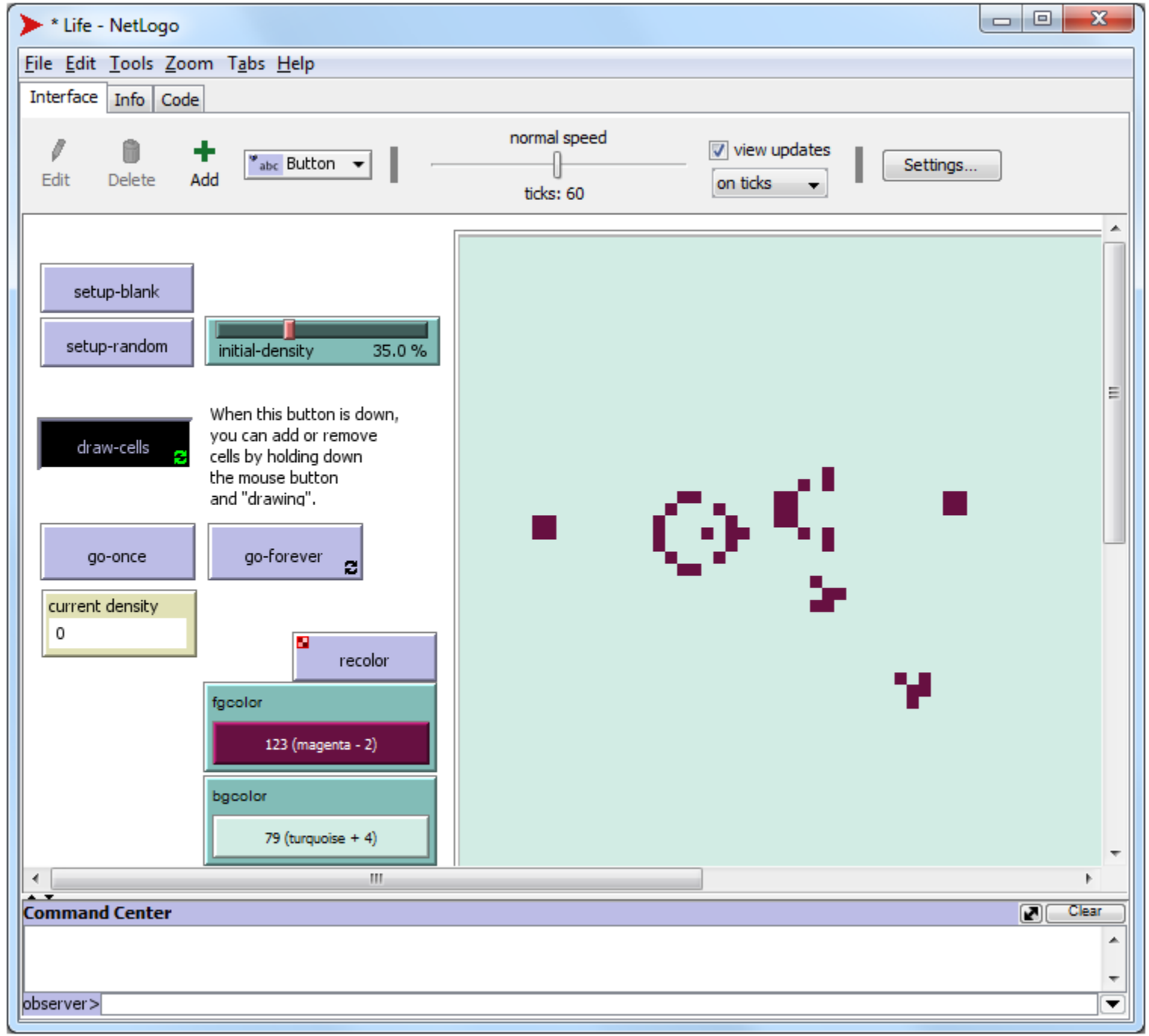


Gosper’s glider gun (wikimedia)



eater,
stopper


* Life - NetLogo
File Edit Tools Zoom Tabs Help
Interface Info Code
Edit
Delete
Add
Button
normal speed
ticks: 0
view updates
on ticks
Settings...
setup-blank
setup-random
initial-density 35.0 %
draw-cells
When this button is down, you can add or remove cells by holding down the mouse button and "drawing".
go-once
go-forever
current density
0
recolor
fgcolor
123 (magenta - 2)
bgcolor
79 (turquoise + 4)
Command Center
Clear
observer>


* Life - NetLogo
File Edit Tools Zoom Tabs Help
Interface Info Code
Edit
Delete
Add
Button
normal speed
ticks: 60
view updates
on ticks
Settings...
setup-blank
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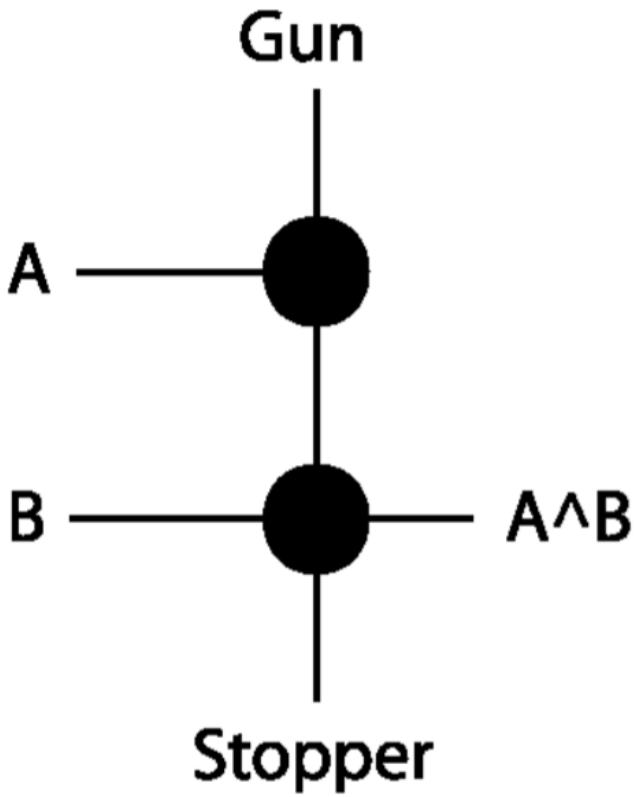
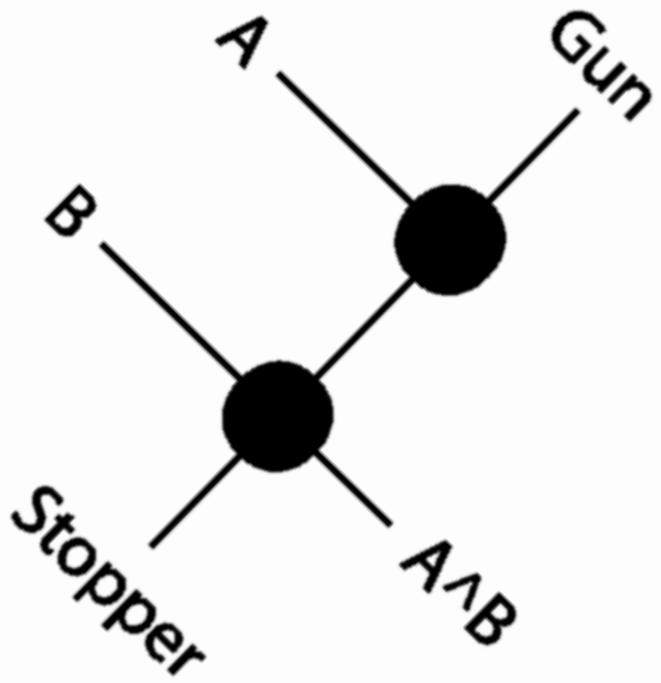
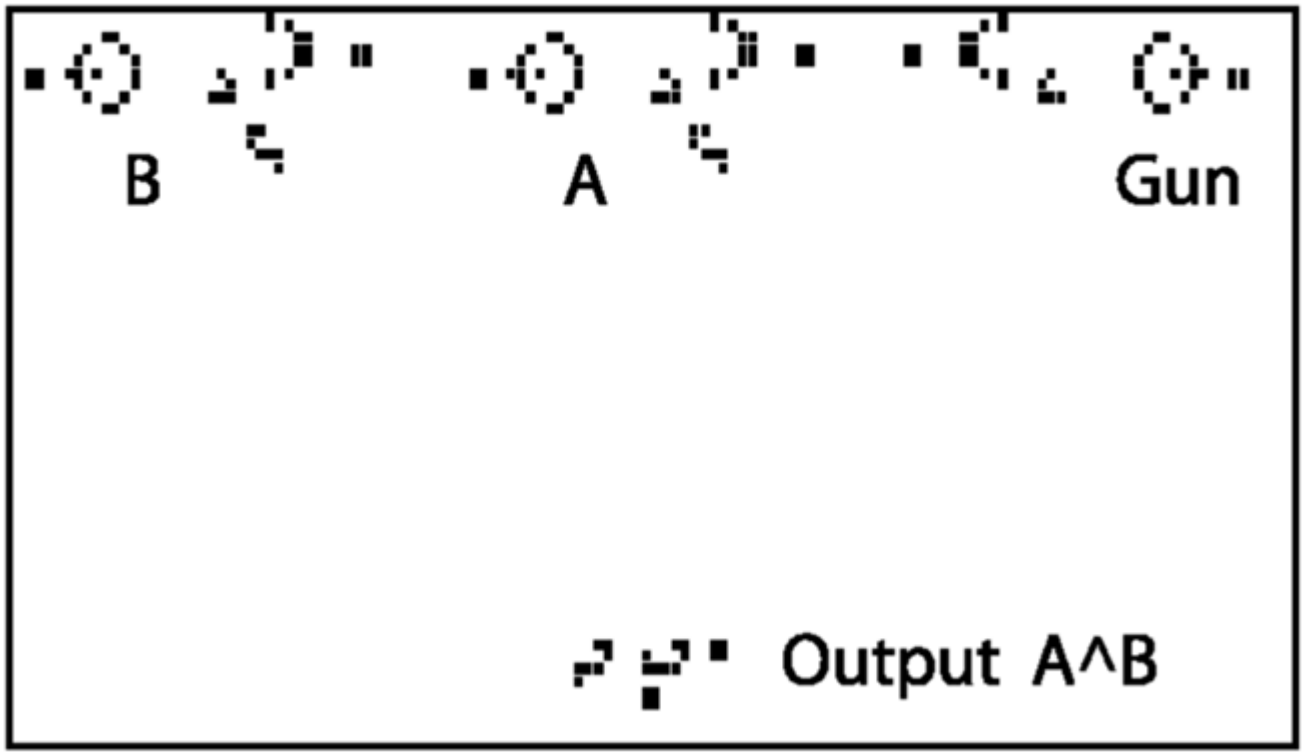
draw-cells
When this button is down, you can add or remove cells by holding down the mouse button and "drawing".
go-once
go-forever
current density
0
recolor
fgcolor
123 (magenta - 2)
bgcolor
79 (turquoise + 4)
Command Center
Clear
observer>

The AND-Gate



Source: Jean Philippe Rennard,
Implementation of Logical functions in the game of life,
Collision-based computing, 2002 - Springer,
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.17.9114&rep=rep1&type=pdf





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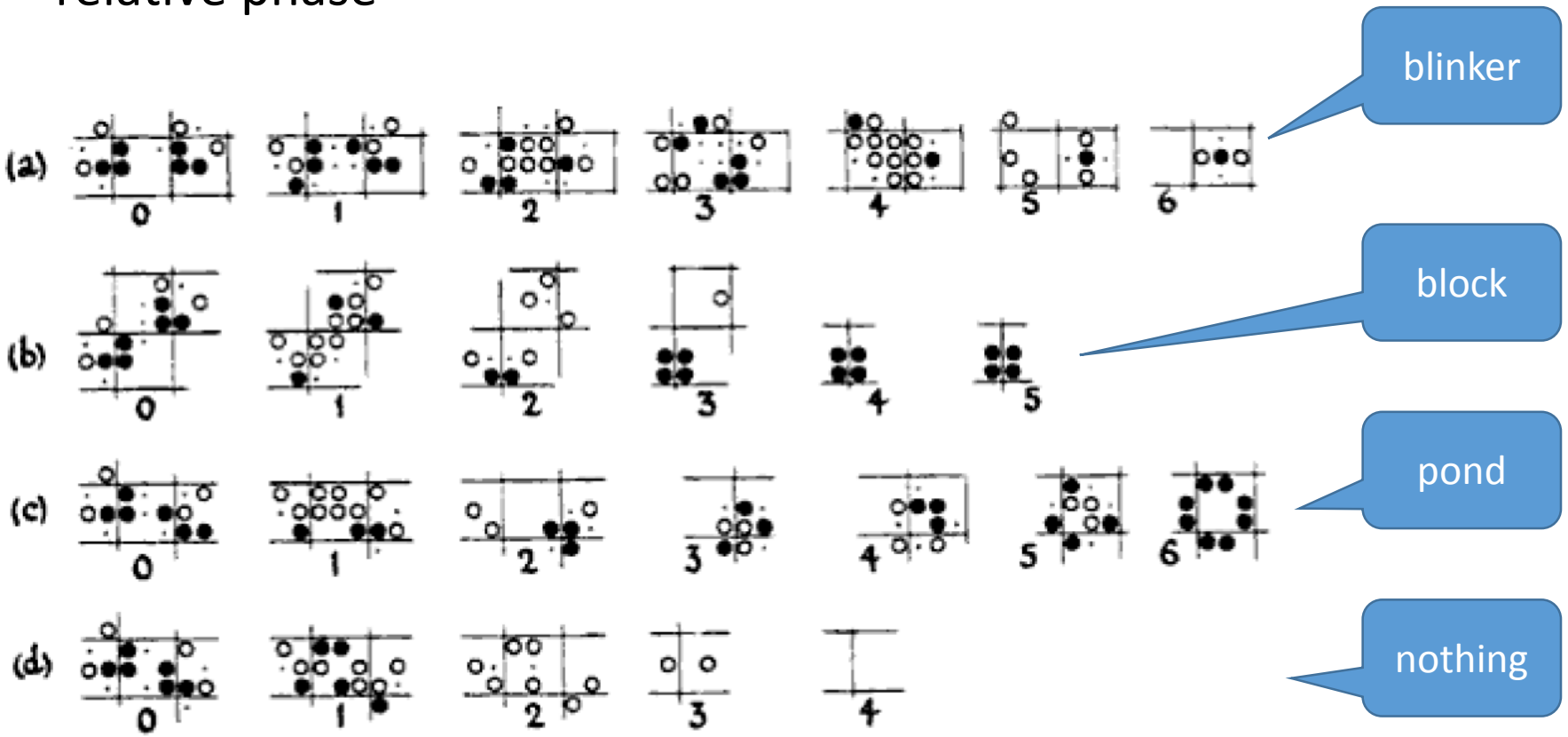
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.17.9114&rep=rep1&type=pdf

Colliding gliders

outcome depends on:

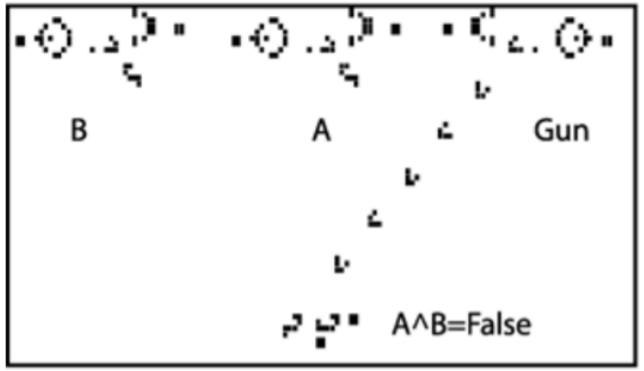
- relative position
- relative phase



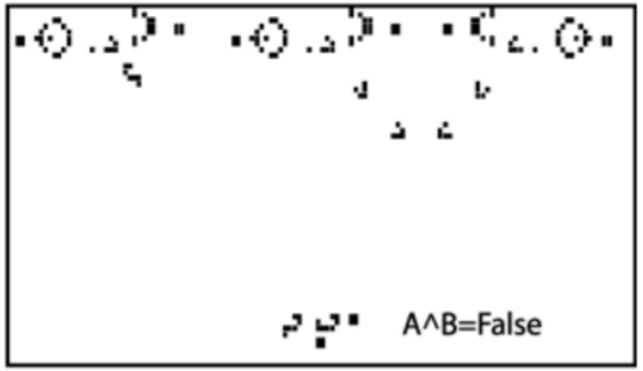
Source: Melissa Gymrek's lecture notes (MIT).
http://web.mit.edu/sp.268/www/2010/lifeSlides.pdf

No glider can activate the output. The result is False.

A False, B False



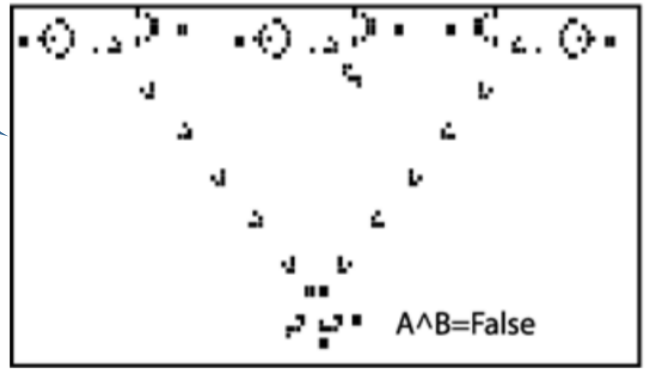
A True, B False



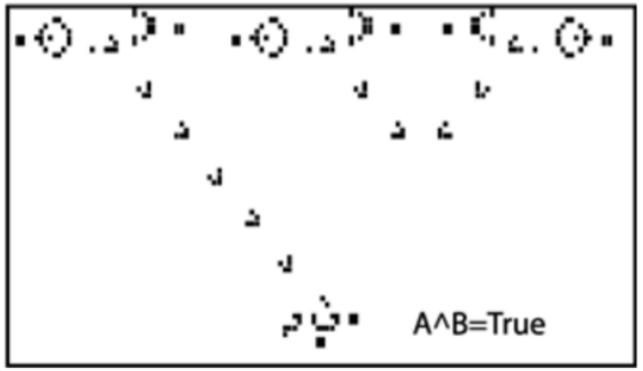
A stops the gun. Since B is false, it cannot activate the output. The result is False.

B cannot join the output since the gun stops it. The result is False.

A False, B True



A True, B True

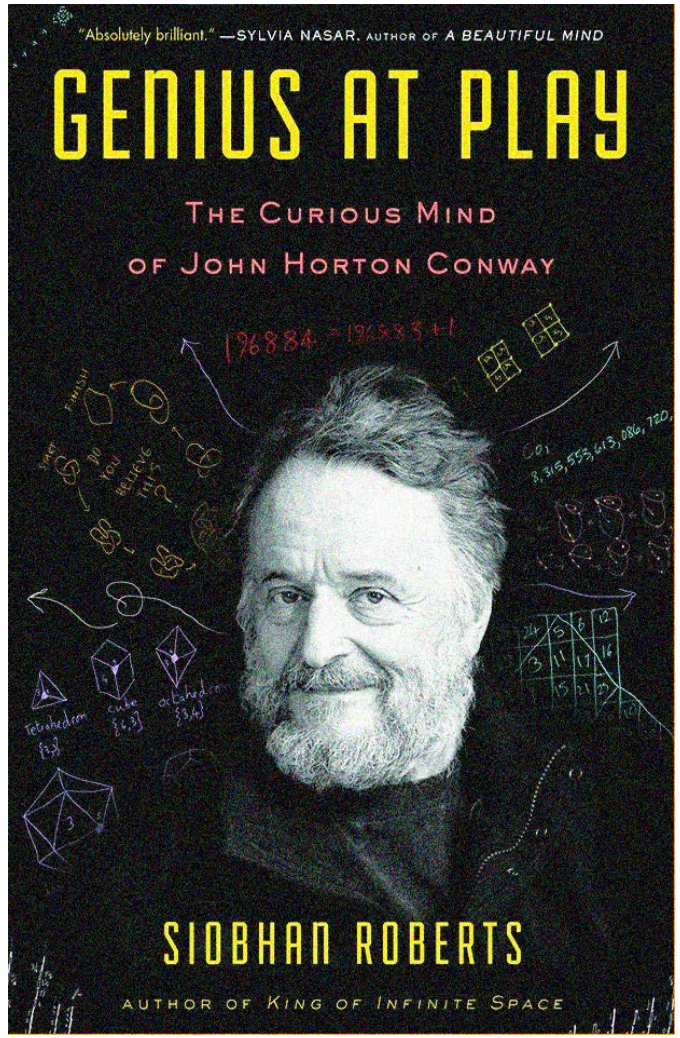


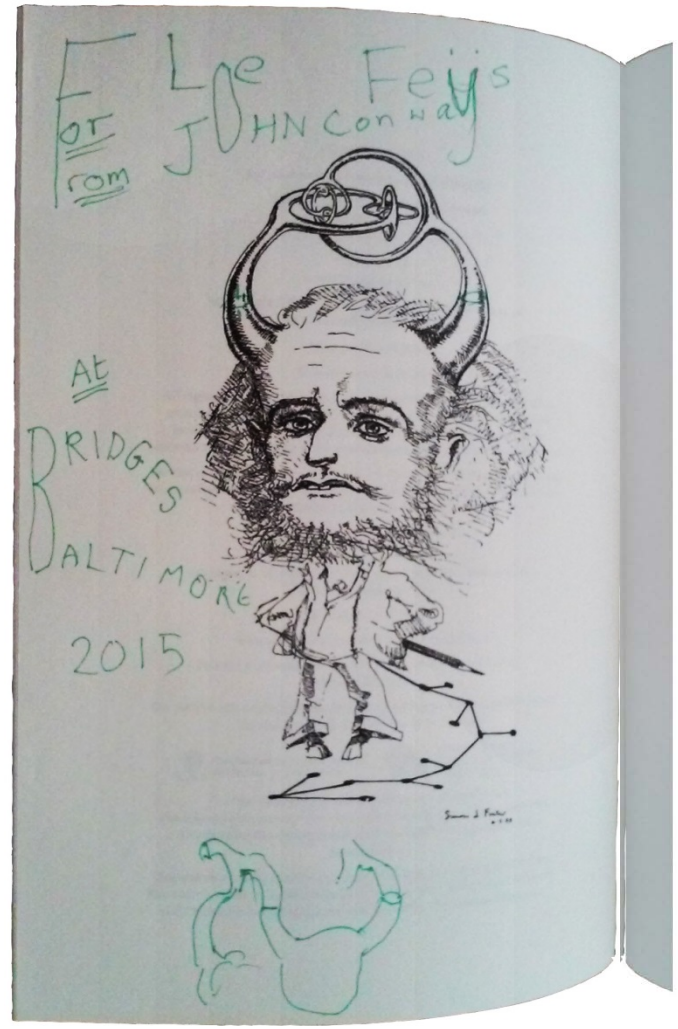
A stops the gun and B can activate the output. The result is True.

Source: Jean Philippe Rennard, Implementation of Logical functions in the game of life, Collision-based computing, 2002 - Springer, http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.17.9114&rep=rep1&type=pdf

Conway’s game of life: conclusions

- There is a large number of life forms, many still to be discovered
- Boolean gates can be simulated
- Even a universal computer can be simulated (Turing machine)

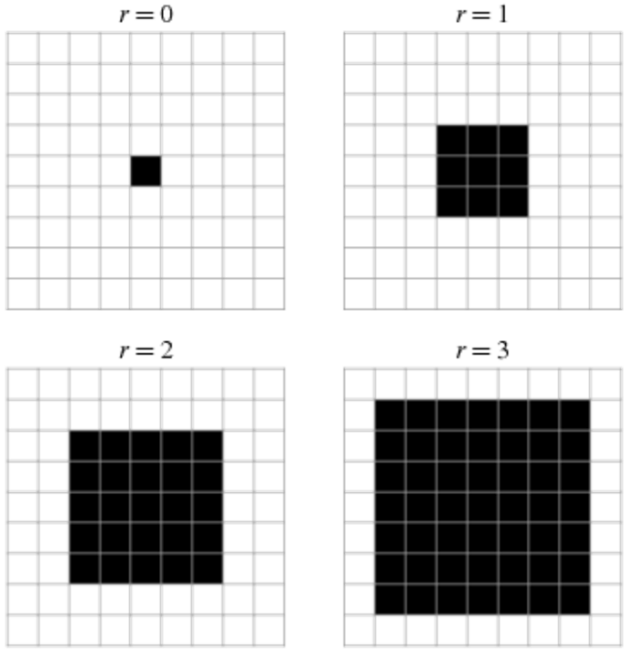




Meeting John Conway at Bridges 2015

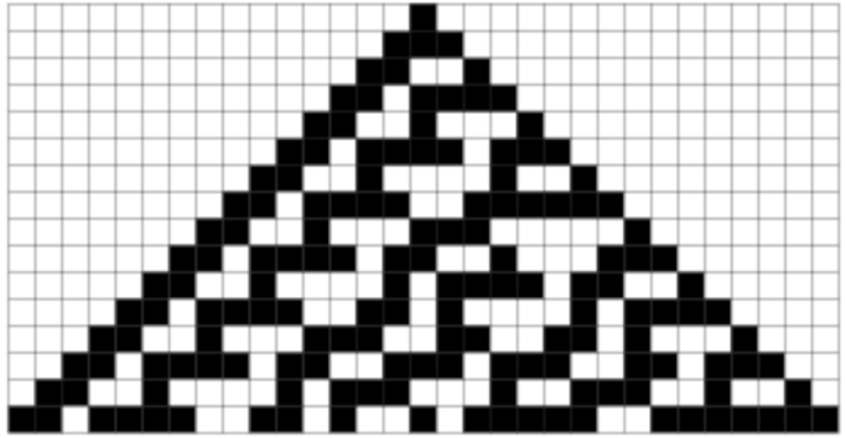
Towards elementary rules: one-dimension, r = 1

Mathworld: Moore neighbourhood (in two dimensions)



rule 30

r=1



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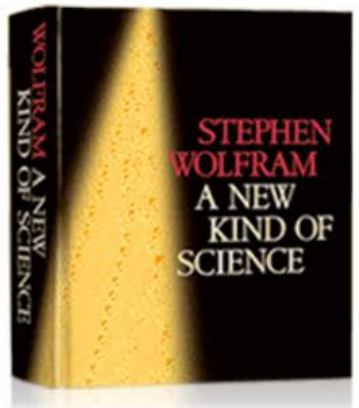

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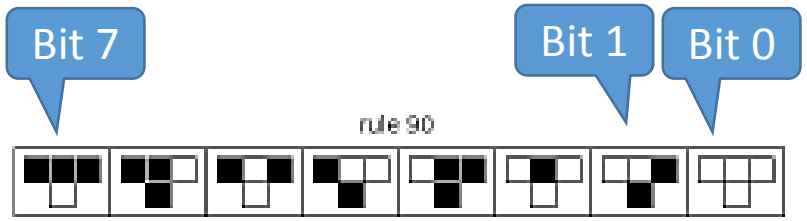
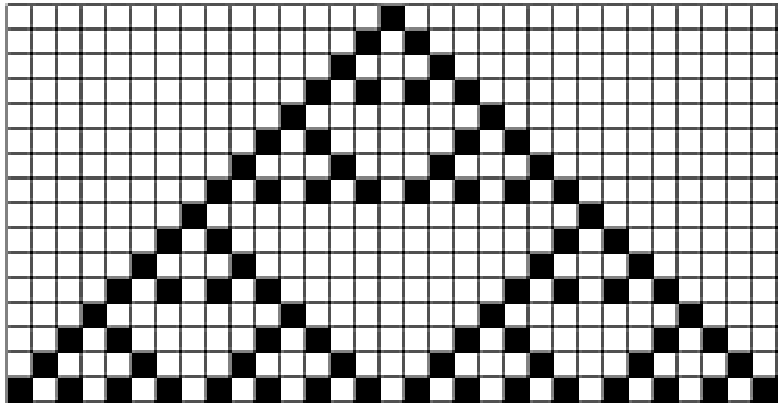
Enumeration scheme for two-color r=1 one-dimensional automata

Example 01011010 (binary) = 64 + 16 + 8 + 2 = 90

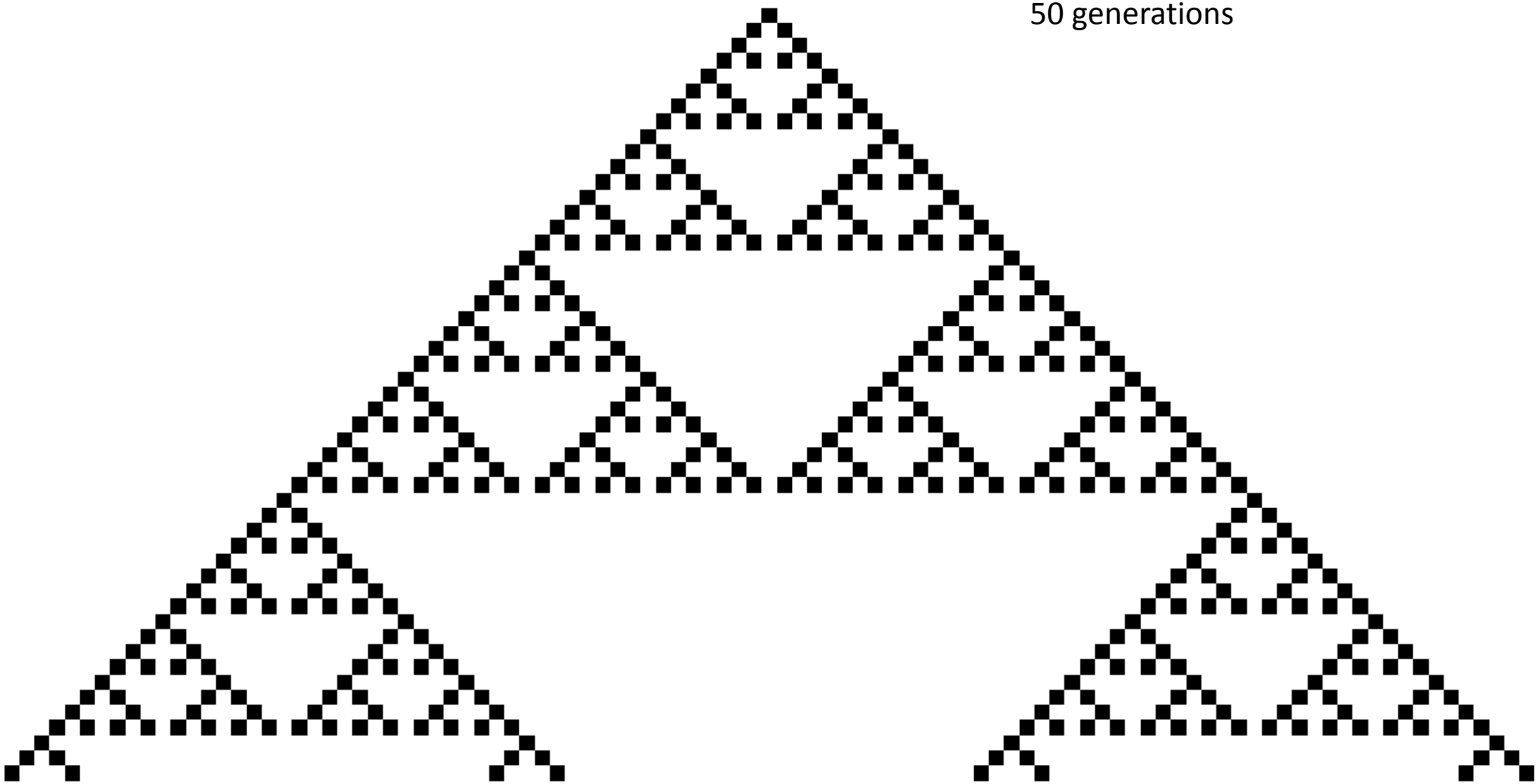




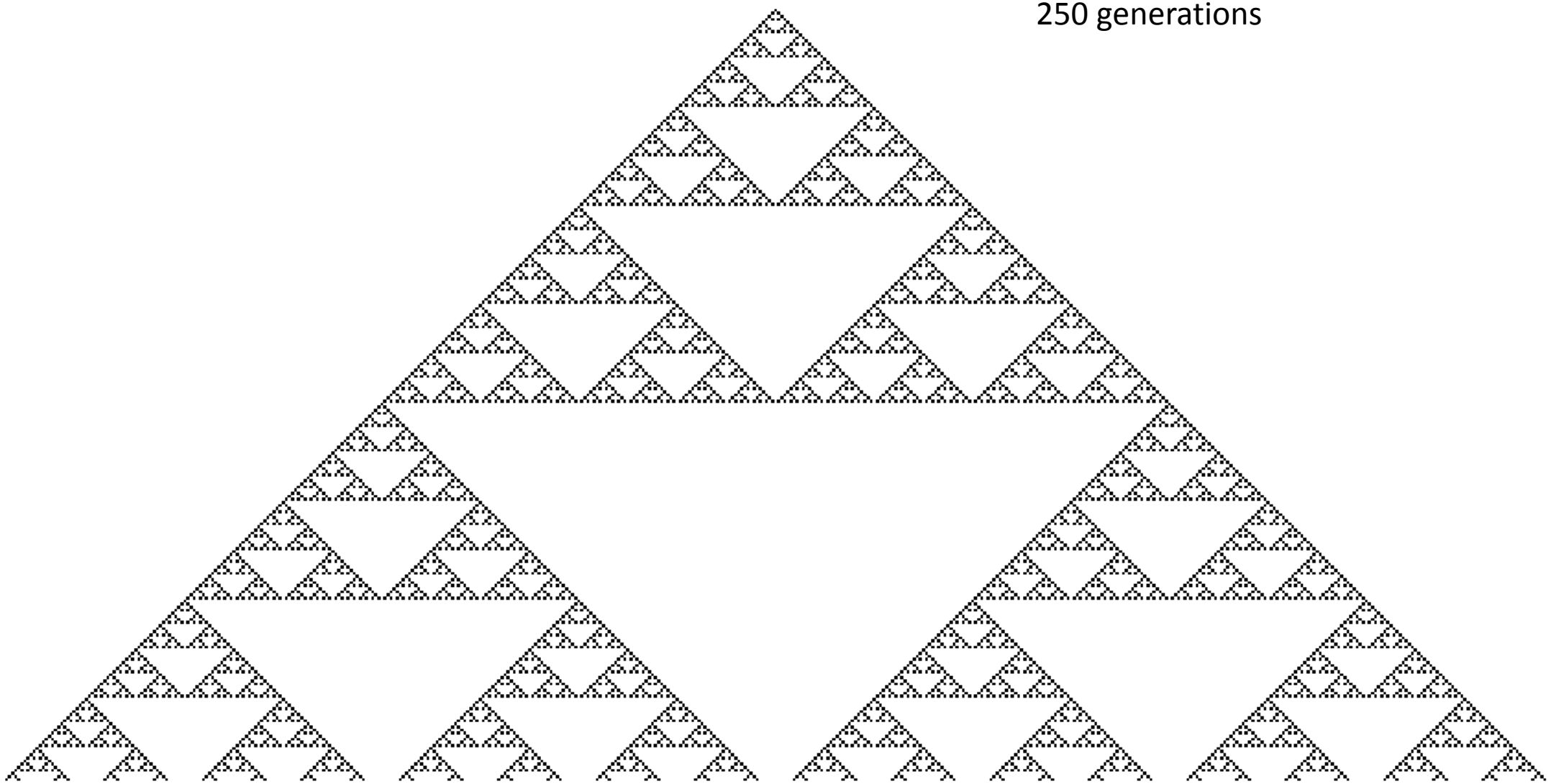
The 2D pattern formed by this rule has fractal dimension log 3 / log 2 ≈ 1.58

Source: NKS

50 generations



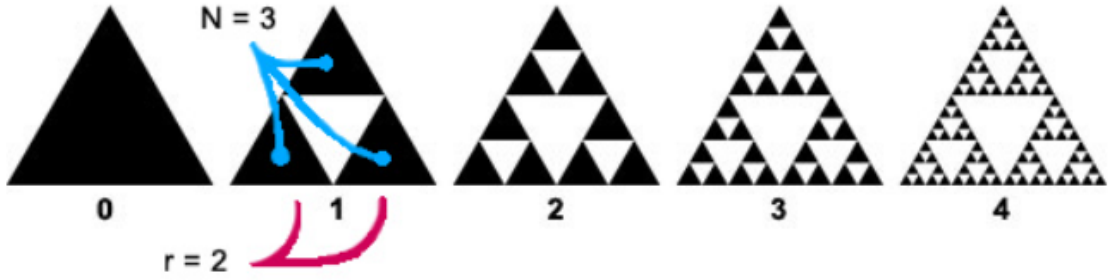
250 generations



This is a so-=called Sierpinski triangle, see https://en.wikipedia.org/wiki/Sierpinski_triangle

Fractal Dimension of the Sierpinski Triangle

Let's use the formula for scaling to determine the dimension of the Sierpinski Triangle fractal. First, take a rough guess at what you might think the dimension will be. Less than 1? Between 1 and 2? Greater than 2? Since the Sierpinski Triangle fits in plane but doesn't fill it completely, its dimension should be less than 2. Let's see if this is true.

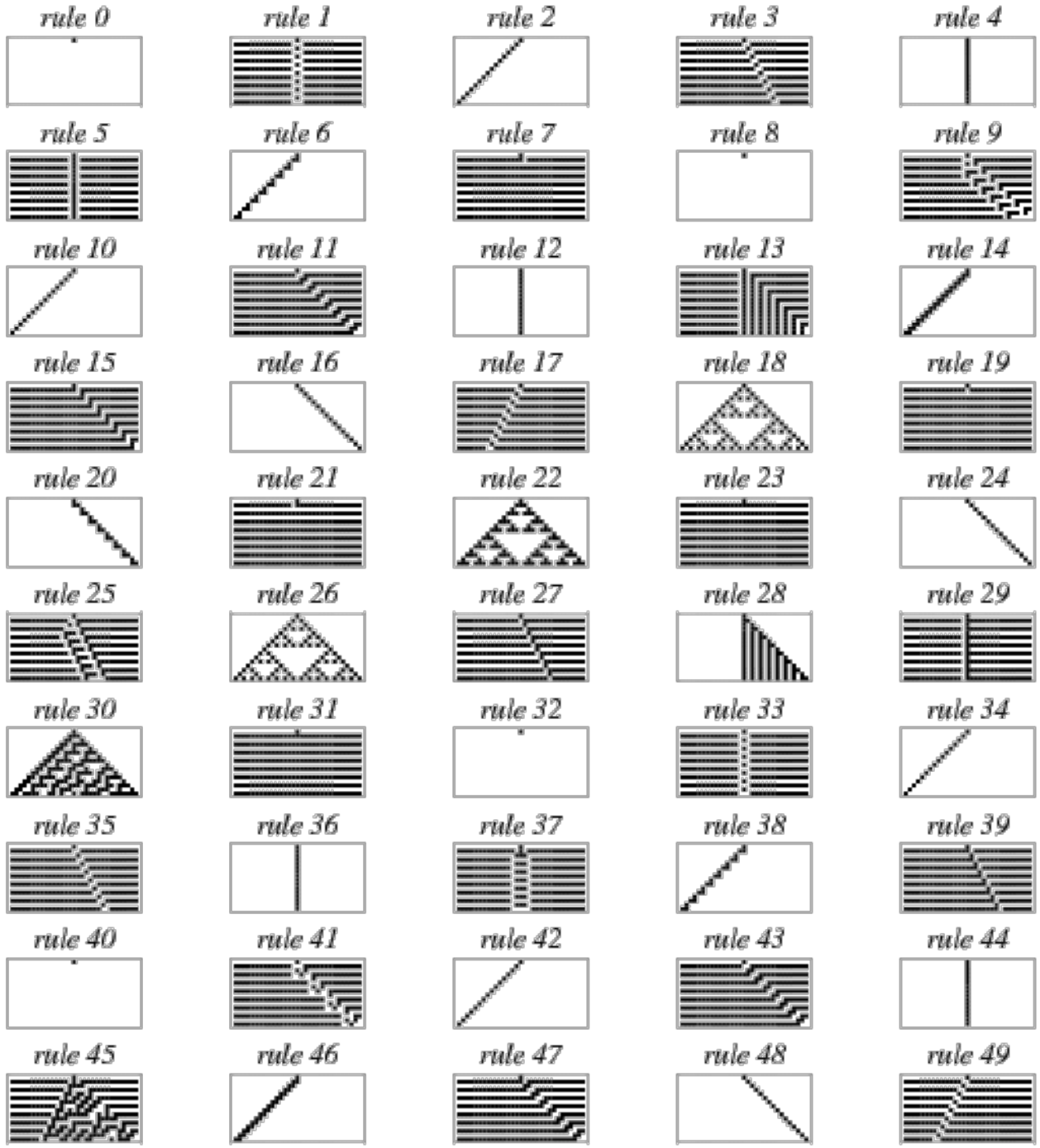


Start with the 0 order triangle in the figure above. The next iteration, order 1, is made up of 3 smaller triangles. And order 2 is made up of 9 triangles. So each iteration of the fractal has 3 times as many triangles, and **N=3**. Next we need to figure out the scaling factor, **r**. How much smaller is each triangle in order 1 than order 0? Look at the edge of each triangle in order 1, and you can see that the edge of each triangle is half the length of the edge of the triangle in order 0. So the scaling factor **r=2**.

That's all we need to know, and we can find the dimension by using the formula:

$$D = \frac{\log(N)}{\log(r)} = \frac{\log(3)}{\log(2)} = 1.585$$

http://fractalfoundation.org/OFC/OFC-10-3.html


rule 0
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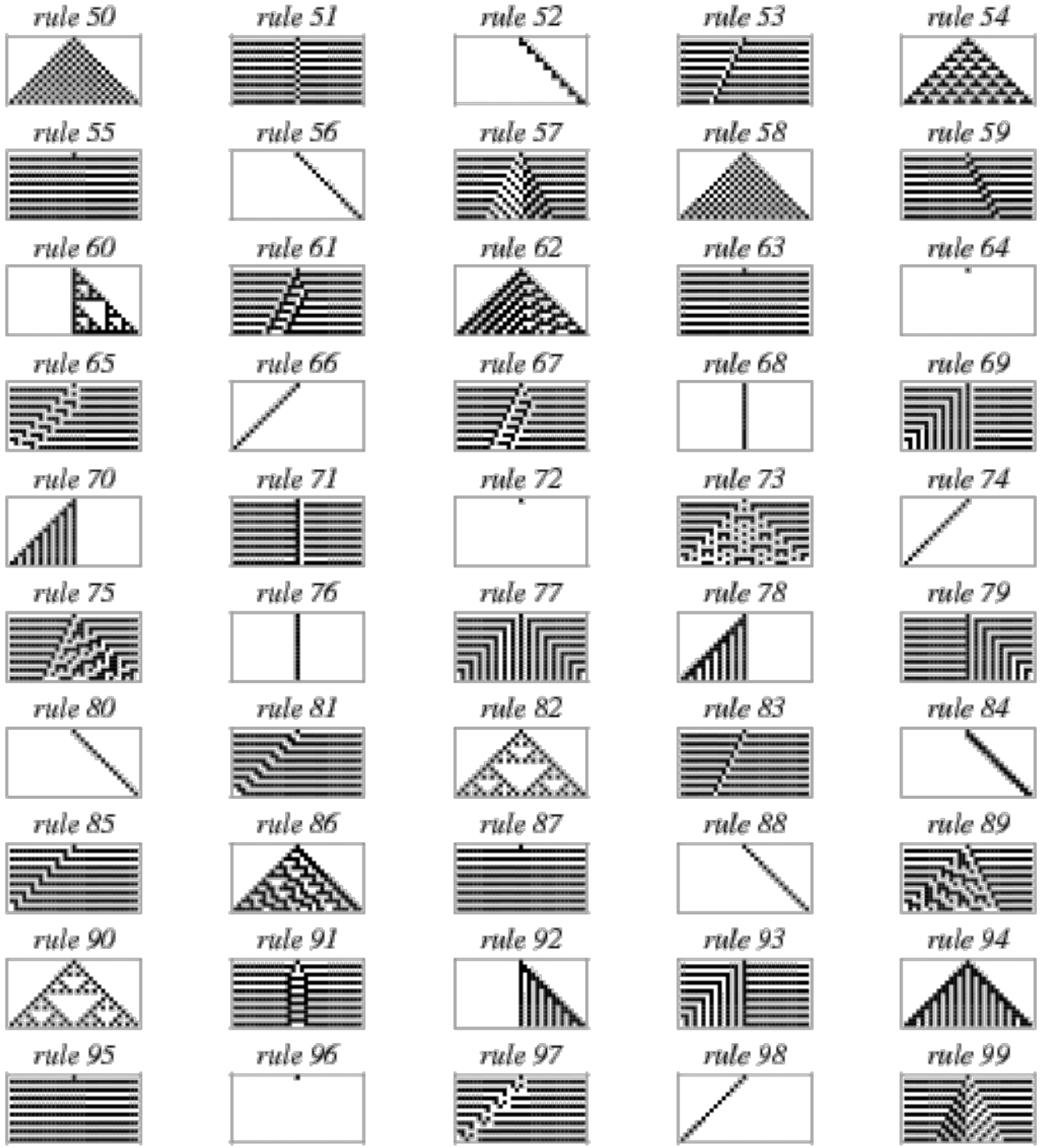
rule 48
rule 49

Source: NKS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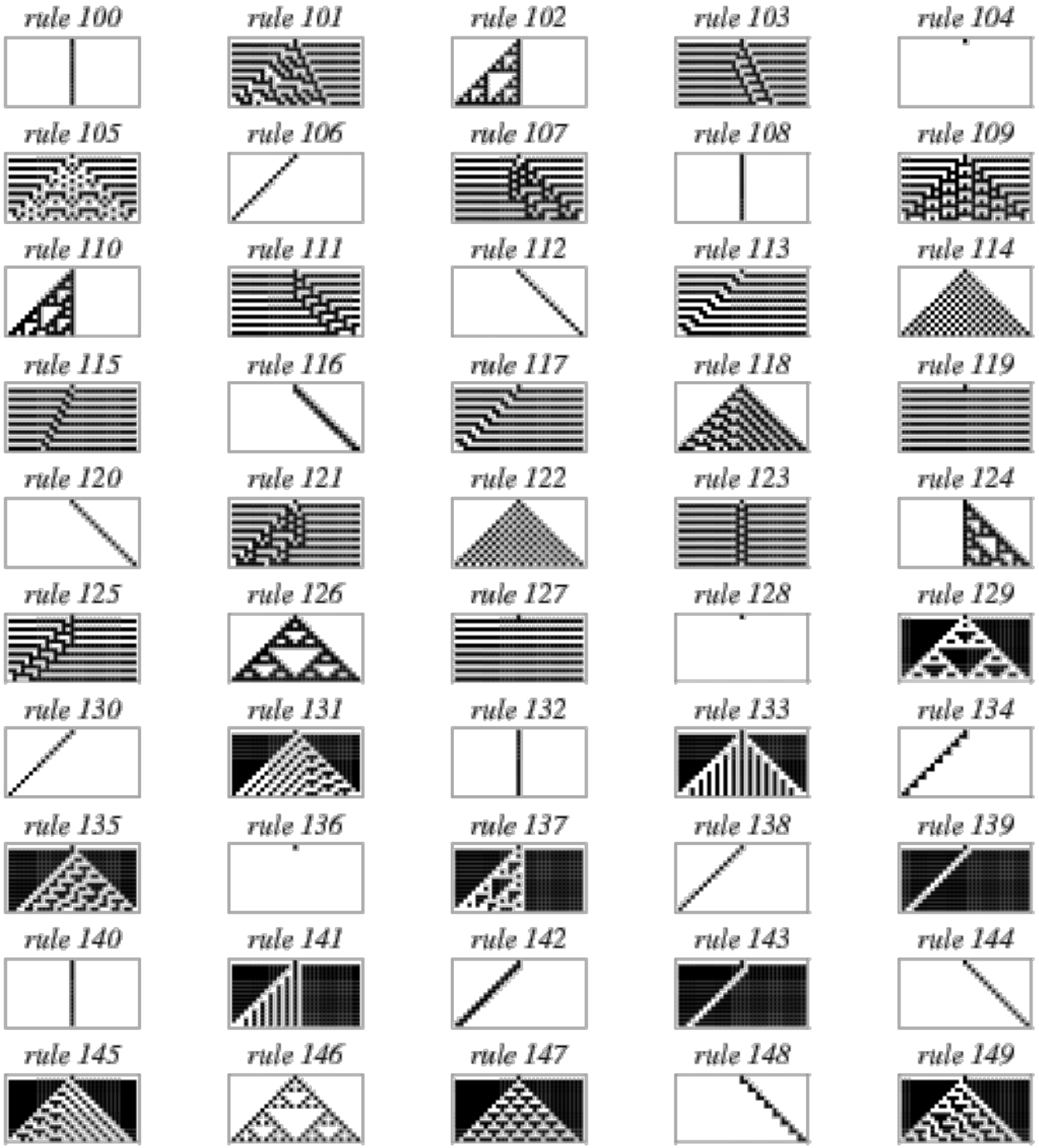
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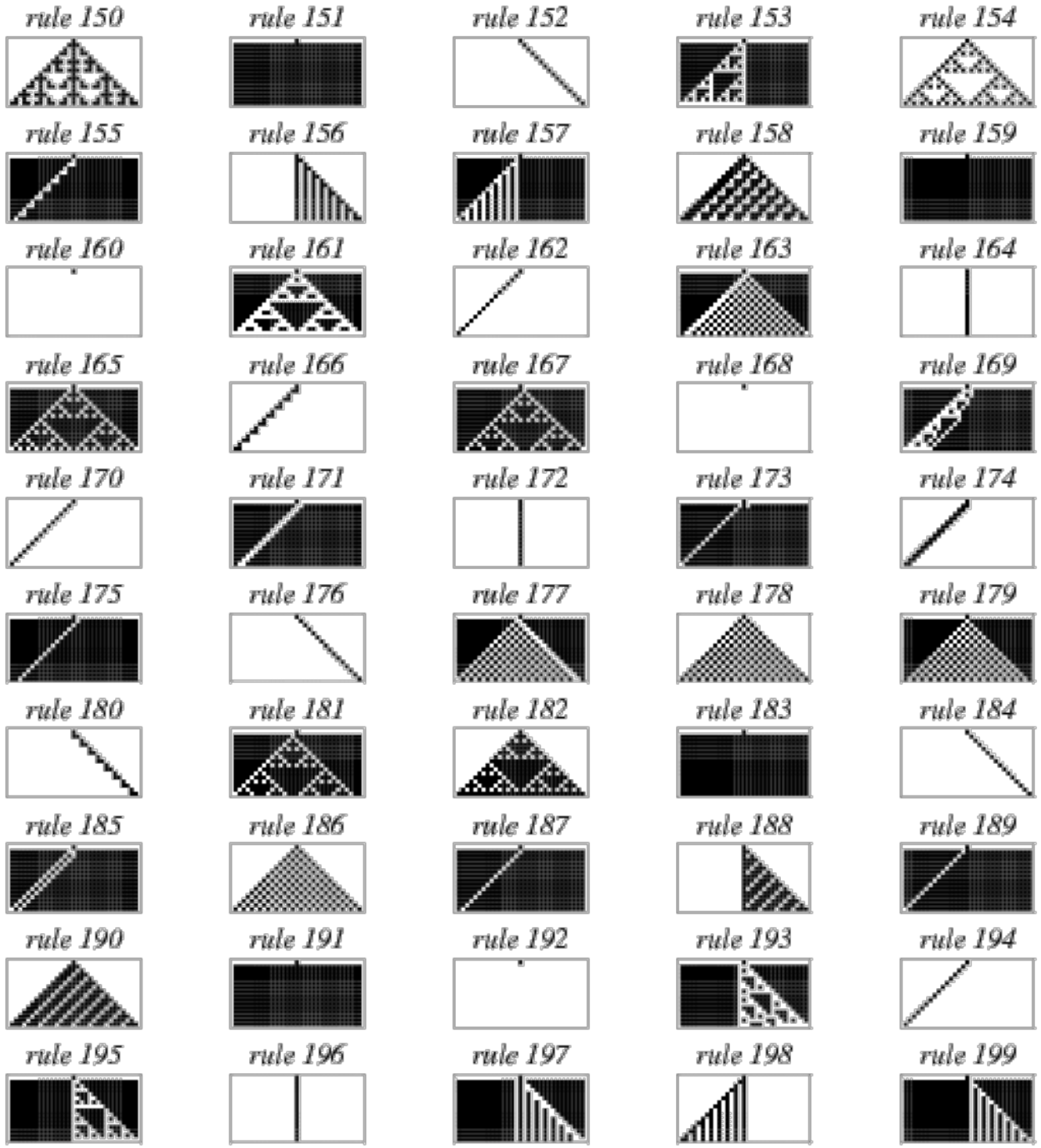
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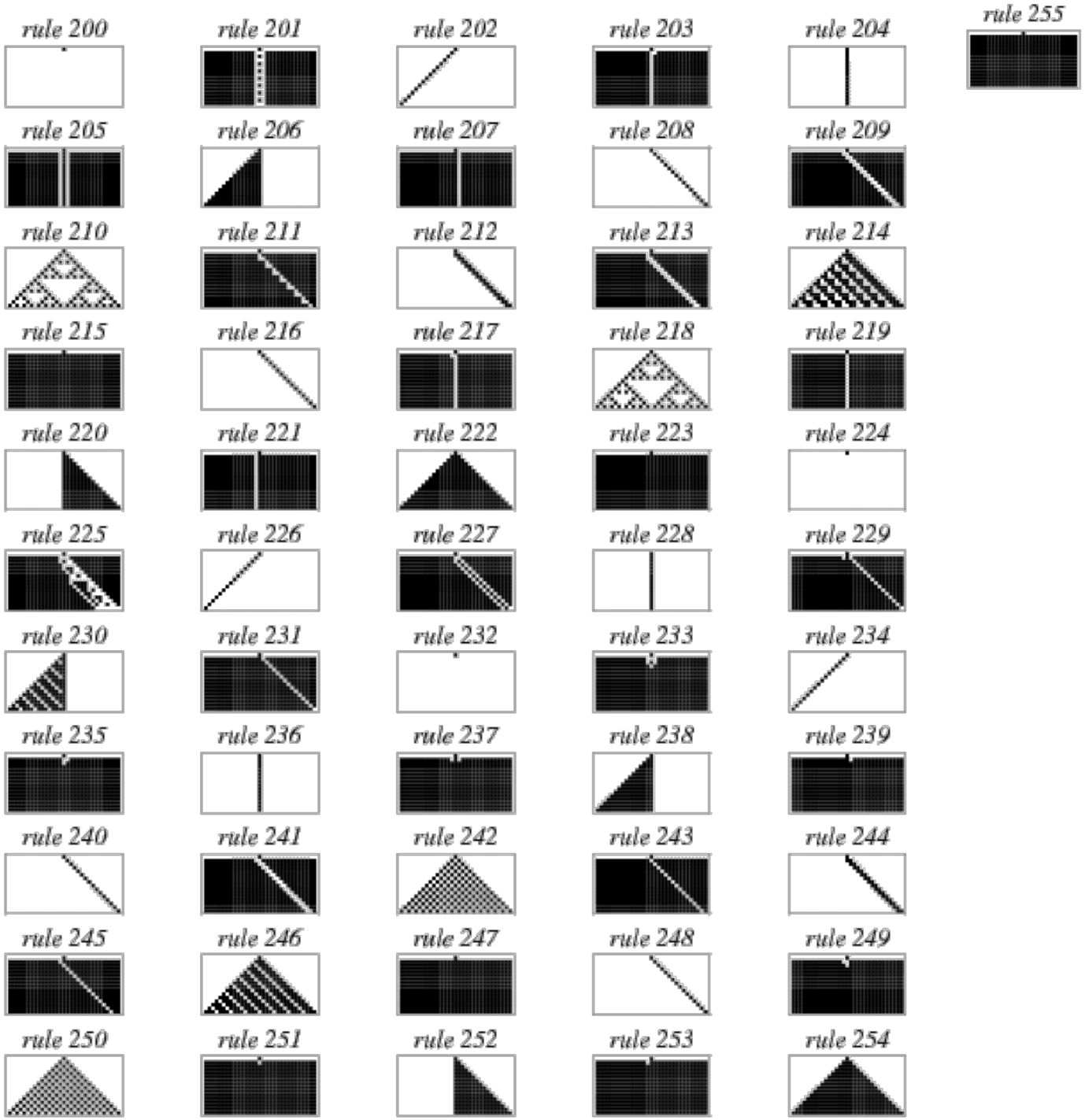
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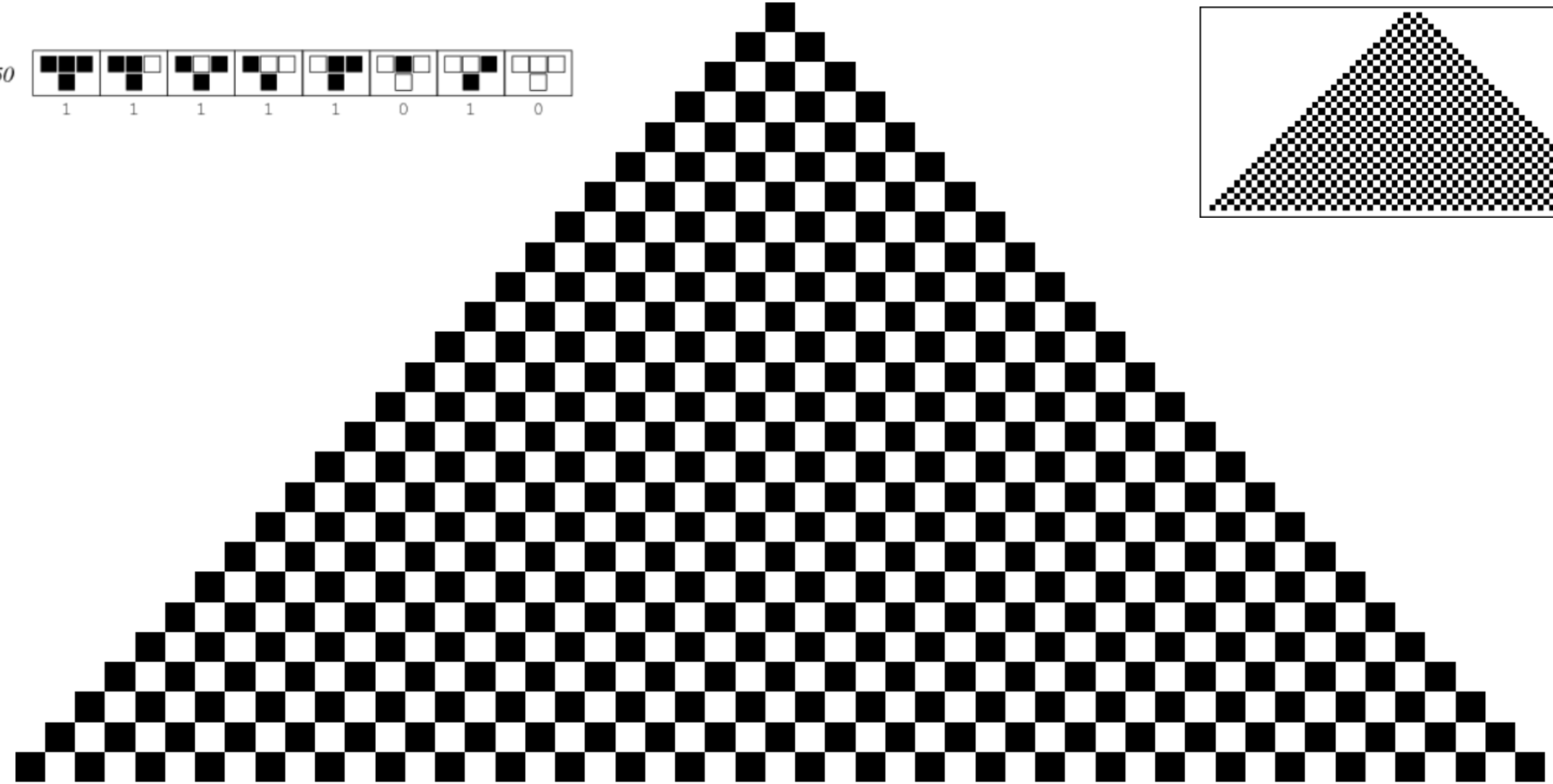
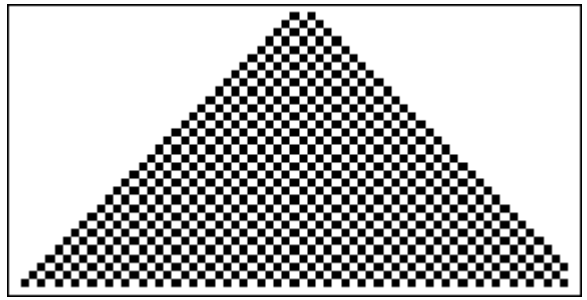
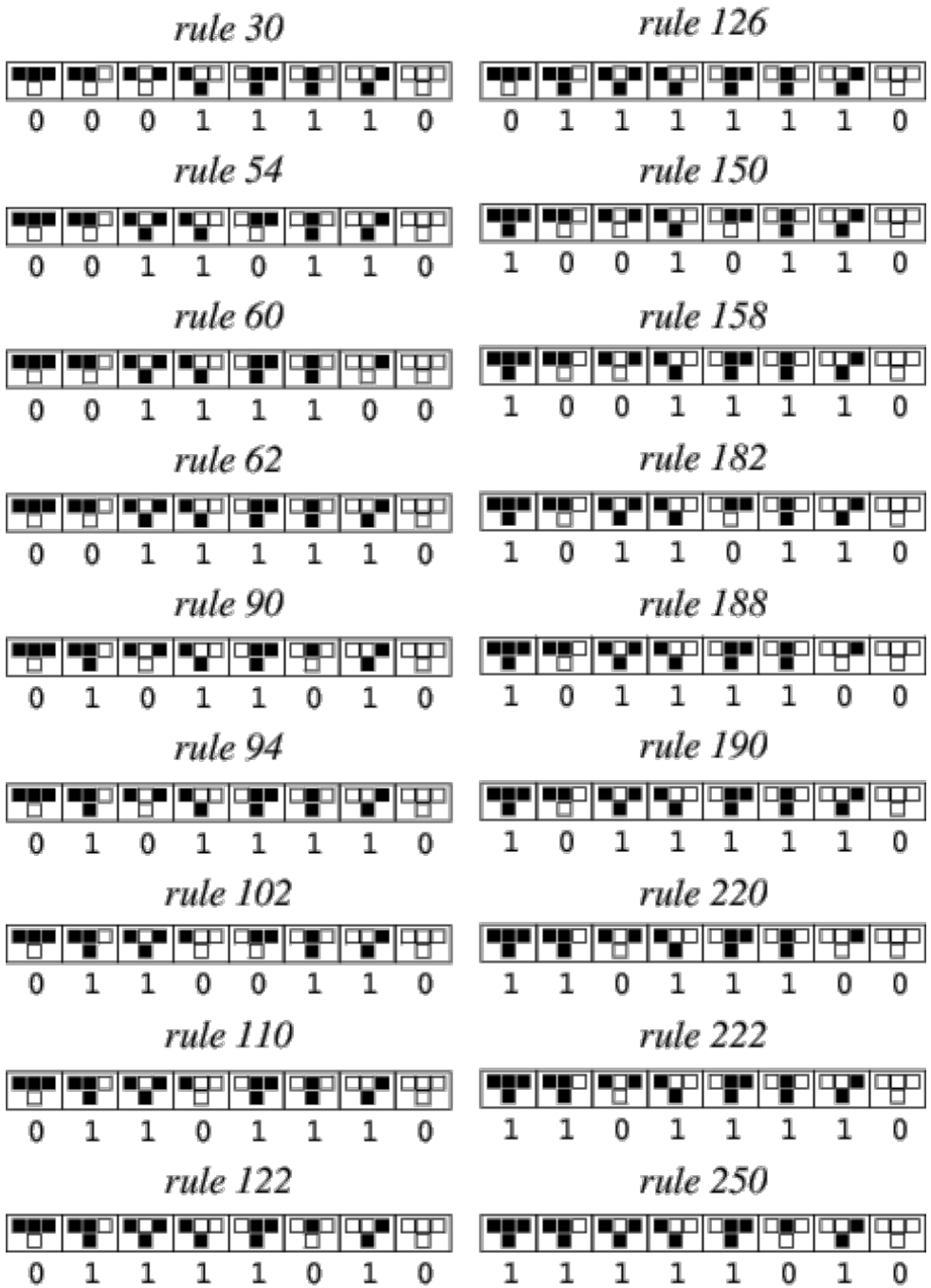
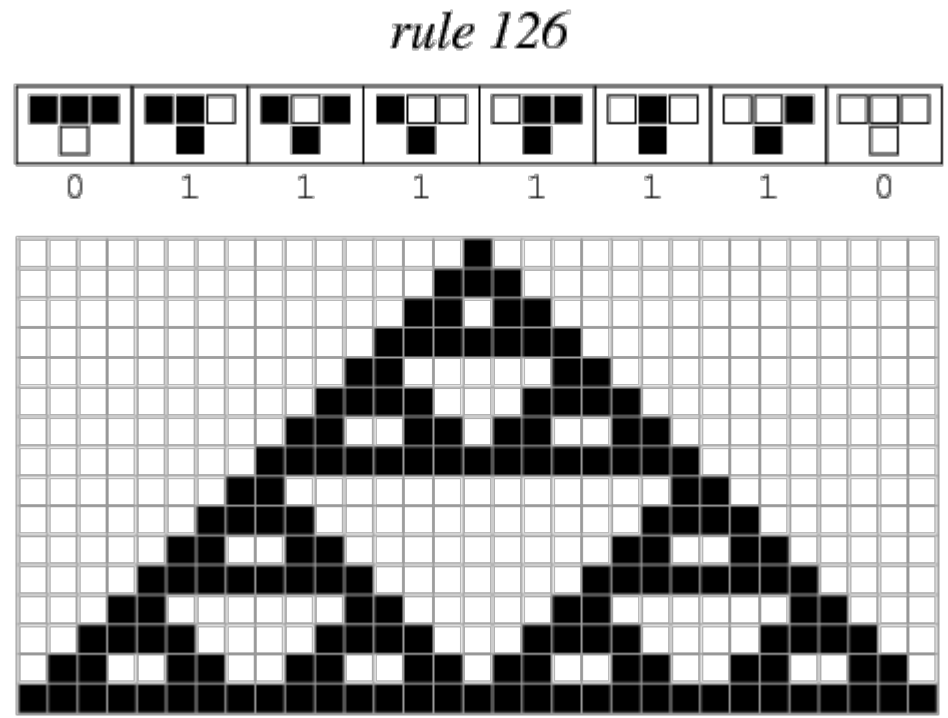
rule 255





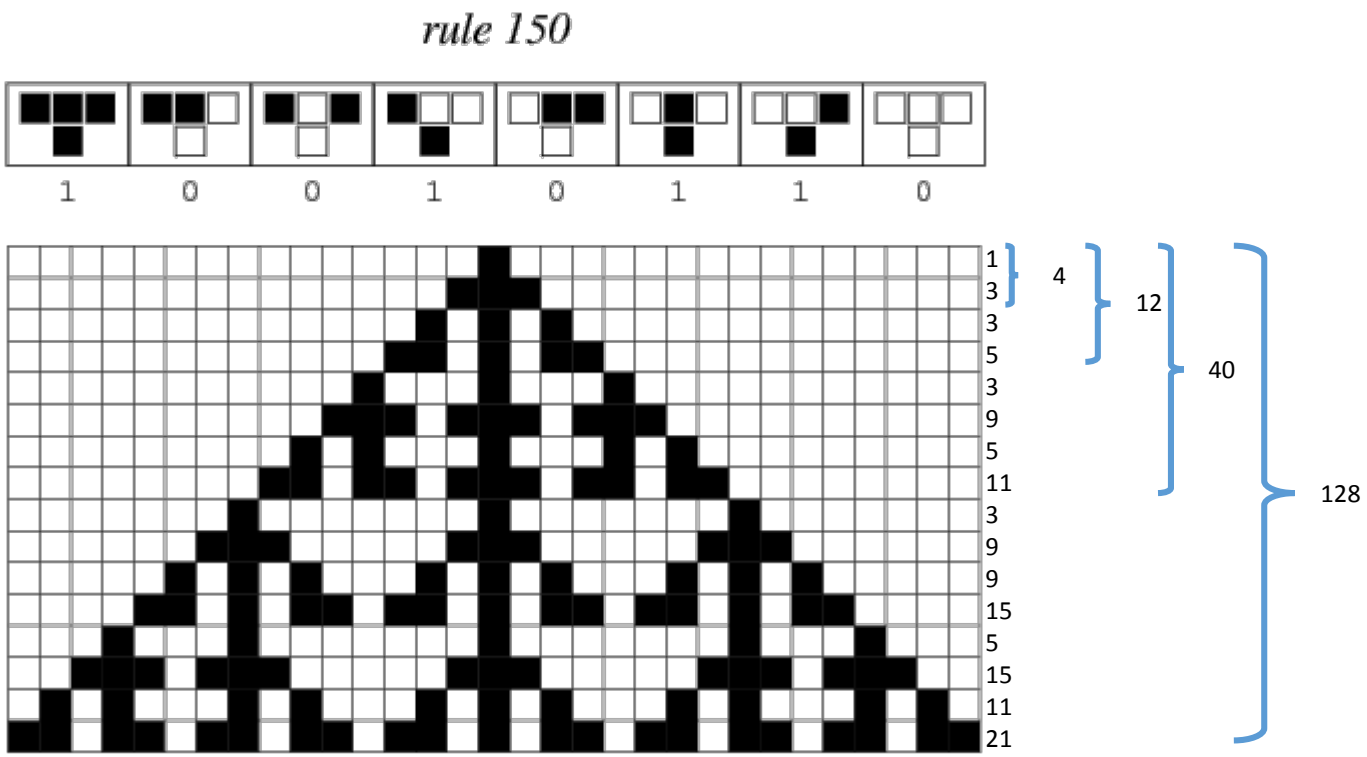
Rule 250 This one is additive, see
http://mathworld.wolfram.com/AdditiveCellularAutomaton.html





Also Rule 90 is “additive”, check Mathworld: http://mathworld.wolfram.com/AdditiveCellularAutomaton.html

source: NKS



D=1 would give 2, 4, 8, 16, …

D=2 would give 2×2, 4×4, 8×8, 16×16, …

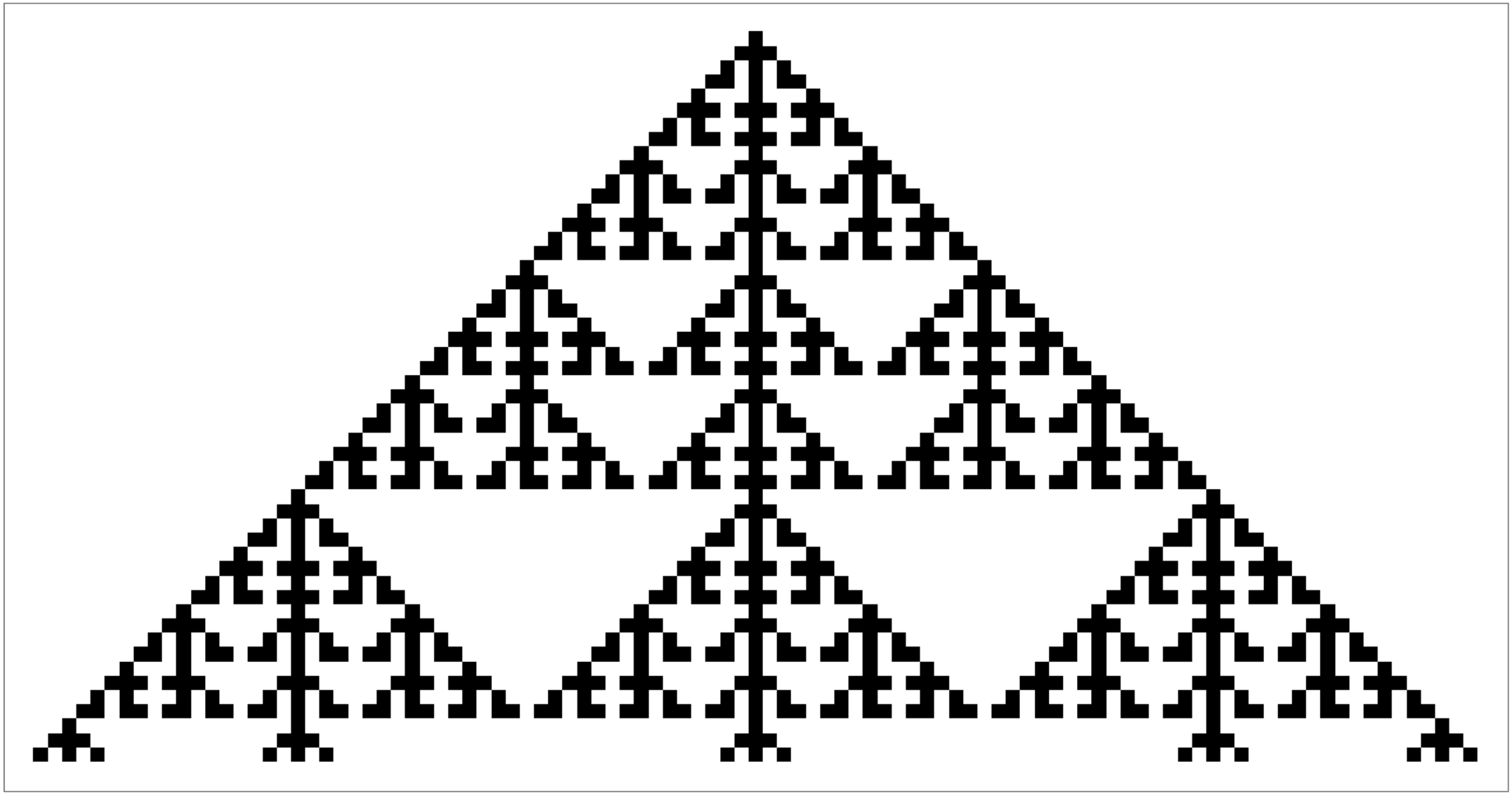
Now we have 2×2, 3×4, 5×8, 8x16, …

Fibonacci numbers: 2,3,5,8,13,21,34, etc.

Limit $F_{n+1}/F_n = (1+\sqrt{5})/2$

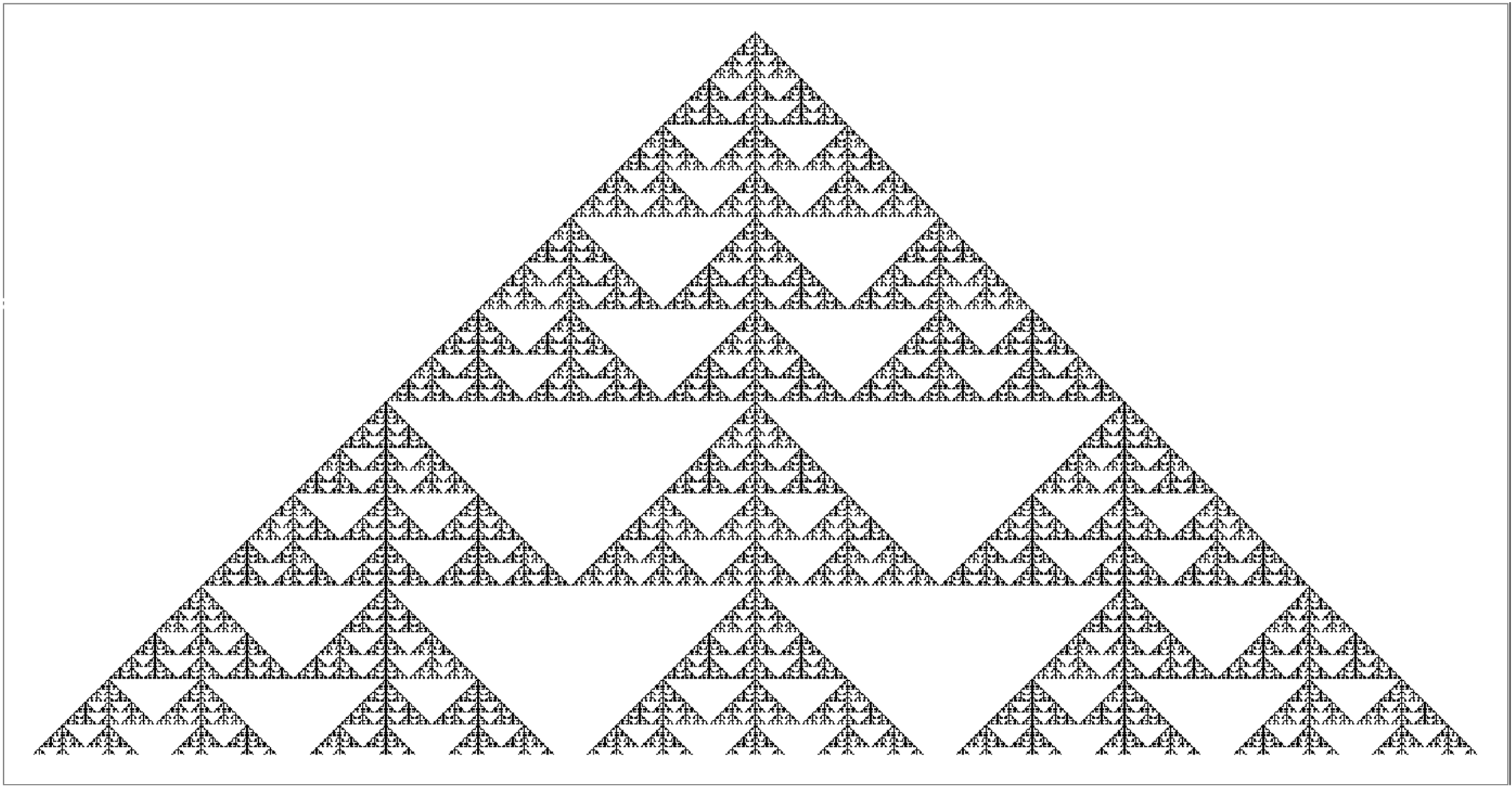
This rule has fractal dimension $\log(1+\sqrt{5}) / \log 2 \approx 1.69$

In[81]:=
```
M = 50;
c = CellularAutomaton[150, {{1}, 0}, M];
ArrayPlot[c]
```

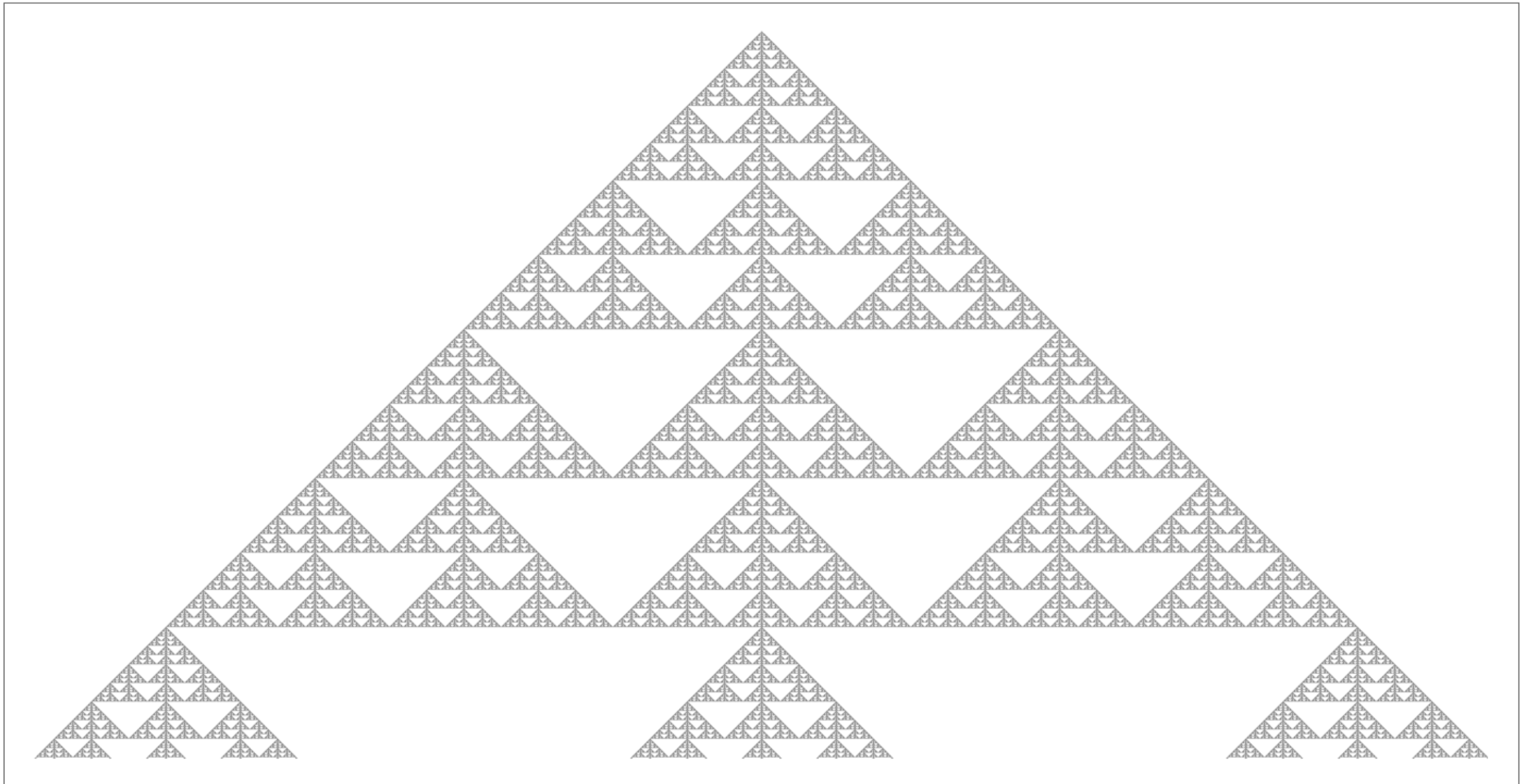


In[78]:=
```
M = 500;
c = CellularAutomaton[150, {{1}, 0}, M];
ArrayPlot[c]
```

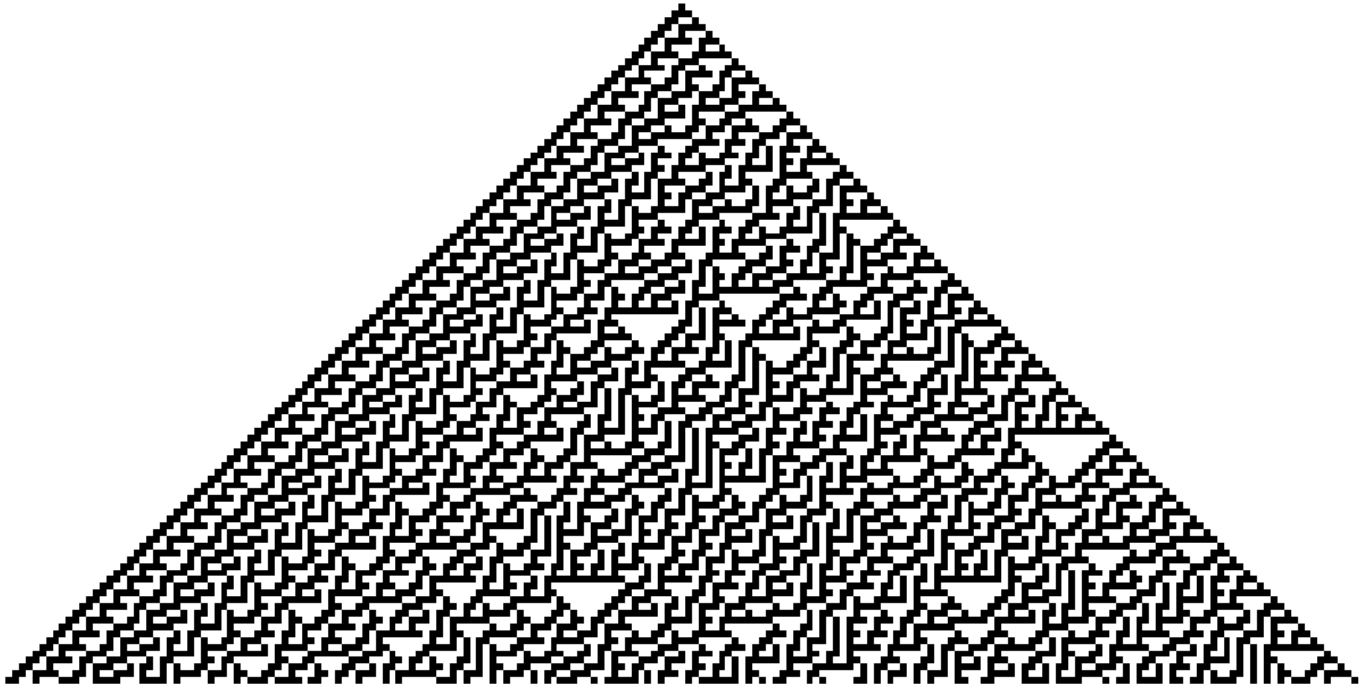
Out[80]=



In[75]:=
```
M = 5000;
c = CellularAutomaton[150, {{1}, 0}, M];
ArrayPlot[c]
```



Rule 30 is chaotic, like a random generator

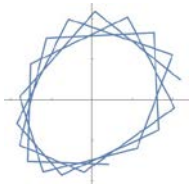


Conus textile by Richard Ling, Wikimedia

An empirical observation is that cellular automata can be classied according to the complexity and information produced by the behavior of the pattern they produce:

- Class 1 : Fixed; all cells converge to a constant black or white set
- Class 2 : Periodic; repeats the same pattern, like a loop
- Class 3 : Chaotic; pseudo-random
- Class 4 : Complex local structures; exhibits behaviors of both class 2 and class 3; with long lived hard to classify structure.

Source: http://www2.math.uu.se/~david/MCS/lecture3.pdf by David_Sumpter

Wolfram’s one-dimensional automata: conclusions

- Some of the automata have complex behavior
- Some rules are perfect random generators (automaton rule 30)
- Even a universal computer can be simulated (automaton rule 110)
- Wolfram believes our whole universe could be just one big cellular automaton