

Mechanics

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Literature

1. **Fenner R.T., Mechanics of Solids.**
2. http://en.wikibooks.org/wiki/Solid_Mechanics
3. **Roark's Formula's for Stress and Strain.**
4. **Pahl & Beitz, Dubbel Taschenbuch fuer den Maschinenbau**
5. **Blanding D.L., Exact constraint machine design using kinematic principles**
6. **Rosielle P.C.J.N., Constructieprincipes, college dictaat Wtb.**
7. **Sanny J., Moebs M., University Physics.**
8. **Groover M.P., Fundamentals of Modern Manufacturing.**
9. **Ashby M.F., Materials Selection in Mechanical Design.**

Why mechanics?



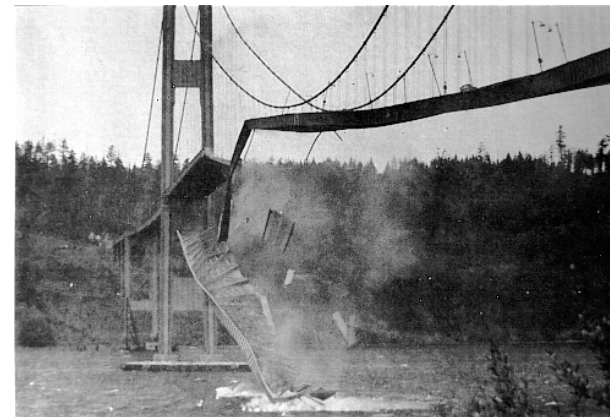
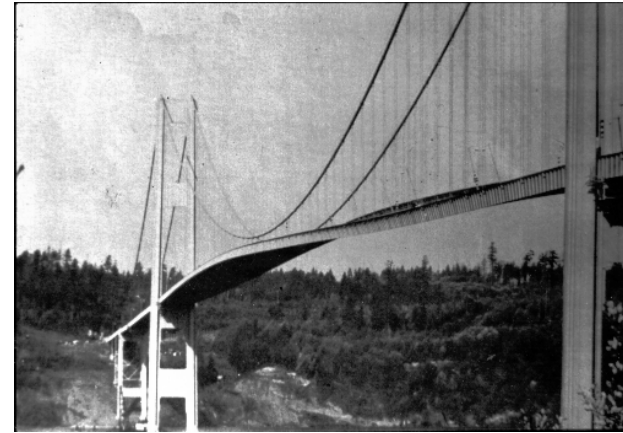
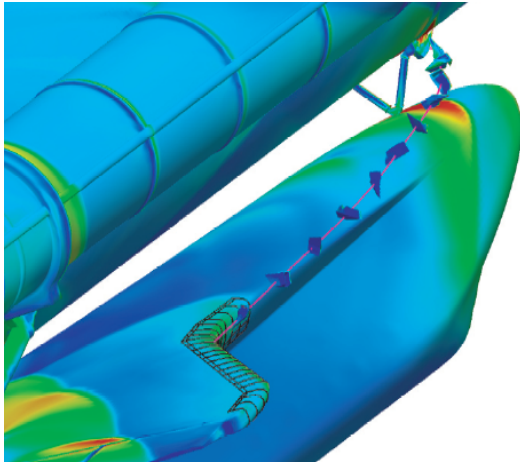
Why mechanics?



To prevent this from happening!



Eschede train crash



Tacoma narrows bridge collapse

If you think knowledge is expensive, try ignorance!

Topics:

- SI Units
- Statics
- Free body diagrams
- Newtons laws of motion
- Joints and boundary conditions
- Elasticity & plasticity
- Second moment of Inertia
- Stresses
 - Tensile
 - Compression
 - Shear
 - Bending
 - Torsion
- Deformations
- Optimal material use

SI Units

- **Basic units**
 - Kg, m, s, A, K, mol, cd
- **Derived units**
 - N (force) in $\text{Kg} * \text{m/s}^2$
 - M (torque) = $\text{N} * \text{m}$ in $\text{Kg} * \text{m}^2/\text{s}^2$

Statics

Newton's laws, Free body diagrams

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Statics

- **Some definitions needed**
 - **Equilibrium**
 - **Scalar**
 - **Vector**
 - **Force**
 - **Moment**
 - **Couple**
- **Examples of these definitions**

Definitions:

- **Equilibrium:**
 - A state of equilibrium is a state of no acceleration
 - In either translational or
 - Rotational senses
- **Scalar:**
 - A quantity having only a magnitude (mass, temperature)

Definitions (cont.)

- **Vector:**

- A quantity having both magnitude and direction (displacement, force)

- **Force:**

- A force is that interaction between bodies which results in an acceleration or a deformation
 - The interaction can occur either through
 - Direct contact of the bodies or
 - Remotely such as gravitation

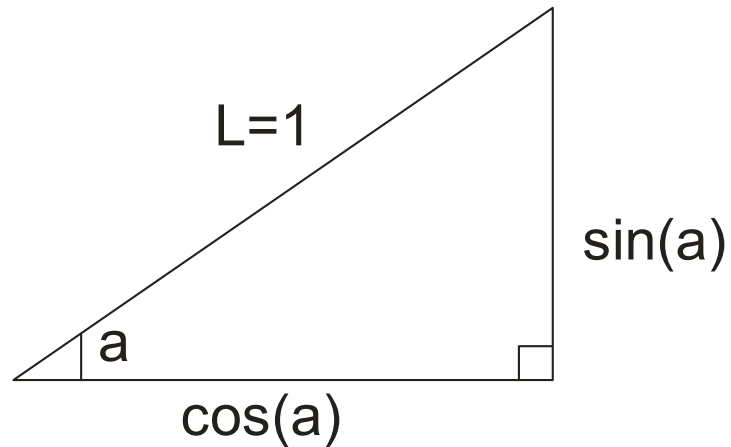
Definitions (cont.)

- **Moment:**
 - A moment is the product of the magnitude of a force and the perpendicular distance of its line of action from a particular point. Moment is also a vector quantity.
- **Couple:**
 - Two forces equal in magnitude but opposite in direction whose lines of action are parallel but not colinear

Newton's laws of motion:

1. If there is no external force or external moment acting on a body, then the body experiences no acceleration
2. An external force acting on a body produces an acceleration in the direction of the force, the force being equal to: $F = m * a$
3. The force exerted by one body B1 on another body B2 is equal in magnitude and opposite in direction to the force exerted by B2 on B1

Sine, cosine and tangens



$$\tan(a) = \frac{\sin(a)}{\cos(a)}$$

$$\sin^2(a) + \cos^2(a) = 1$$

Free body diagrams & statically determinate systems

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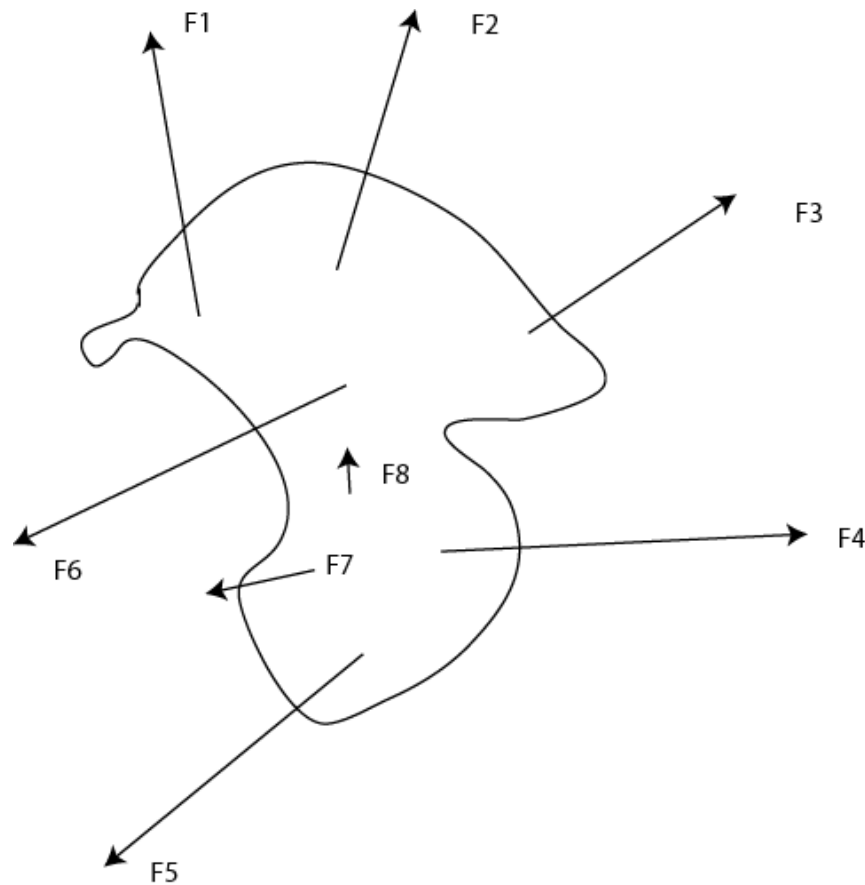
Free body diagrams

- No accelerations then:

$$\sum_{i=1}^n \vec{F}_i = \vec{0}$$

$$\sum_{i=1}^n \vec{M}_i = \vec{0}$$

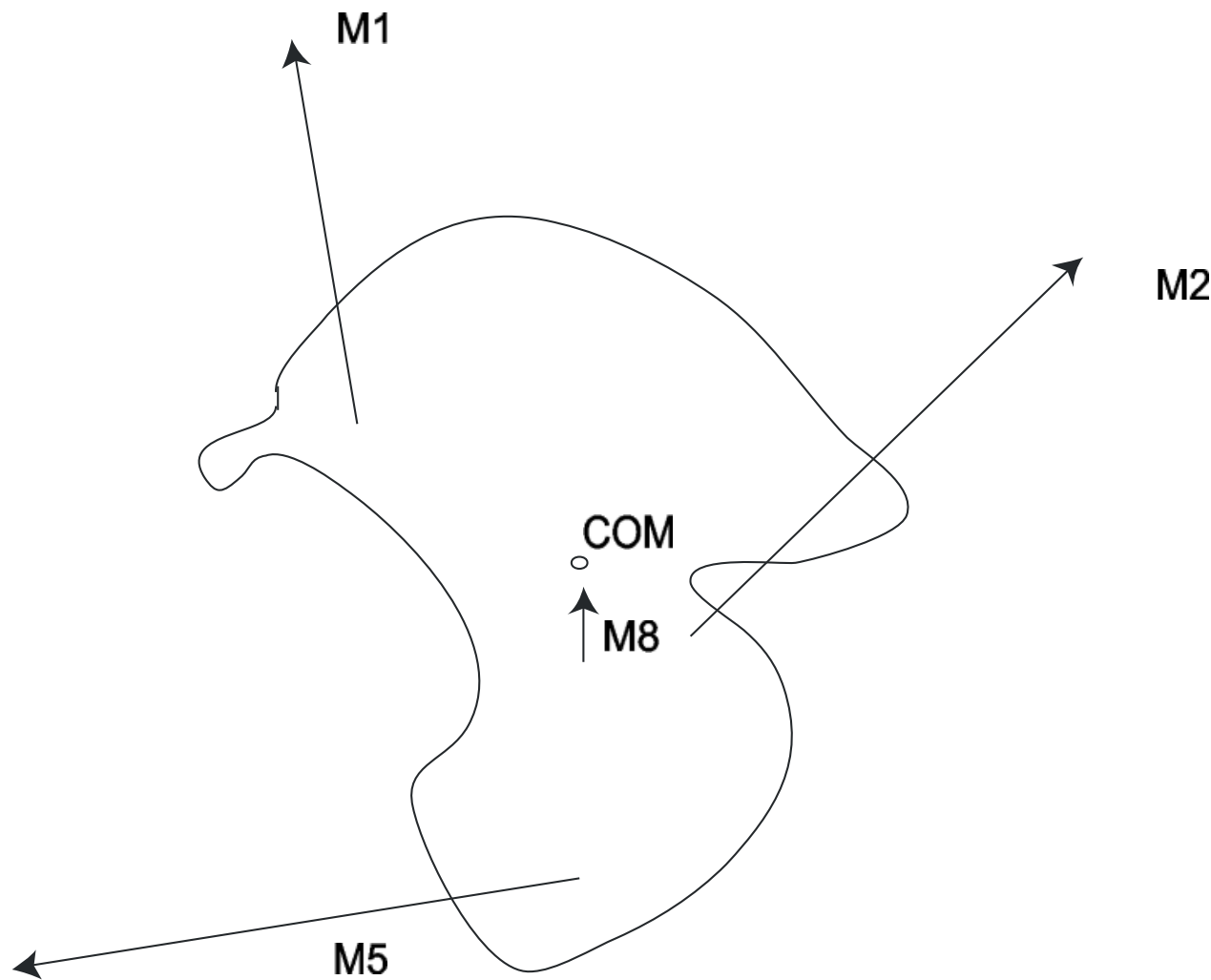
Free body diagrams forces



$$\sum_{i=1}^n \vec{F}_i = \vec{0}$$

$$\sum_{i=1}^n \vec{M}_i = \vec{0}$$

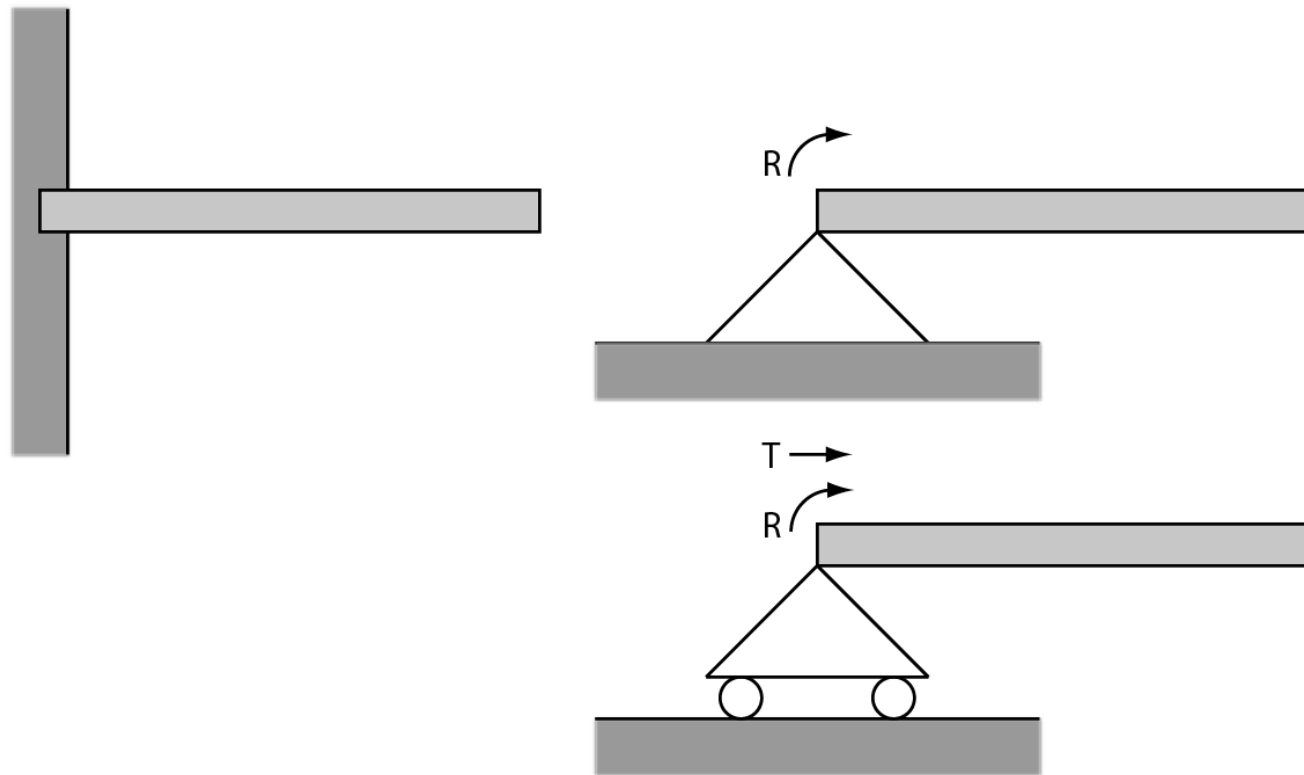
Free body diagrams moments



$$\sum_{i=1}^n \vec{F}_i = \vec{0}$$

$$\sum_{i=1}^n \vec{M}_i = \vec{0}$$

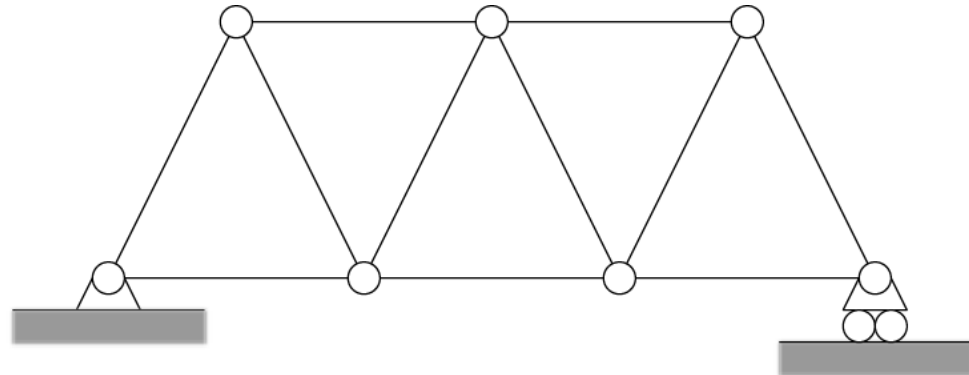
Joints and boundary conditions



Statically Determinate Systems

- **Examples**

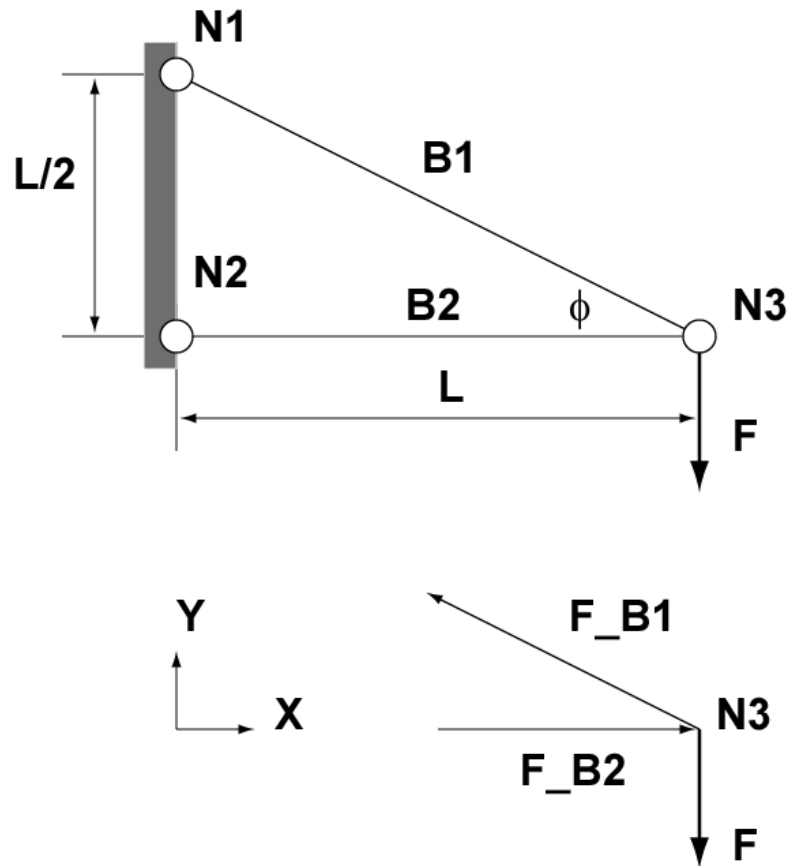
- **Simple bridge**



- **Erasmus bridge**



Example: Free body diagram what are the beam forces?



$$\text{Node 3: } \sum_i \vec{F}_i = \vec{0}$$

N3: y-direction:

$$-F + F_{b1} \sin \phi = 0$$

$$\sin \phi = \frac{L/2}{\sqrt{(L/2)^2 + L^2}}$$

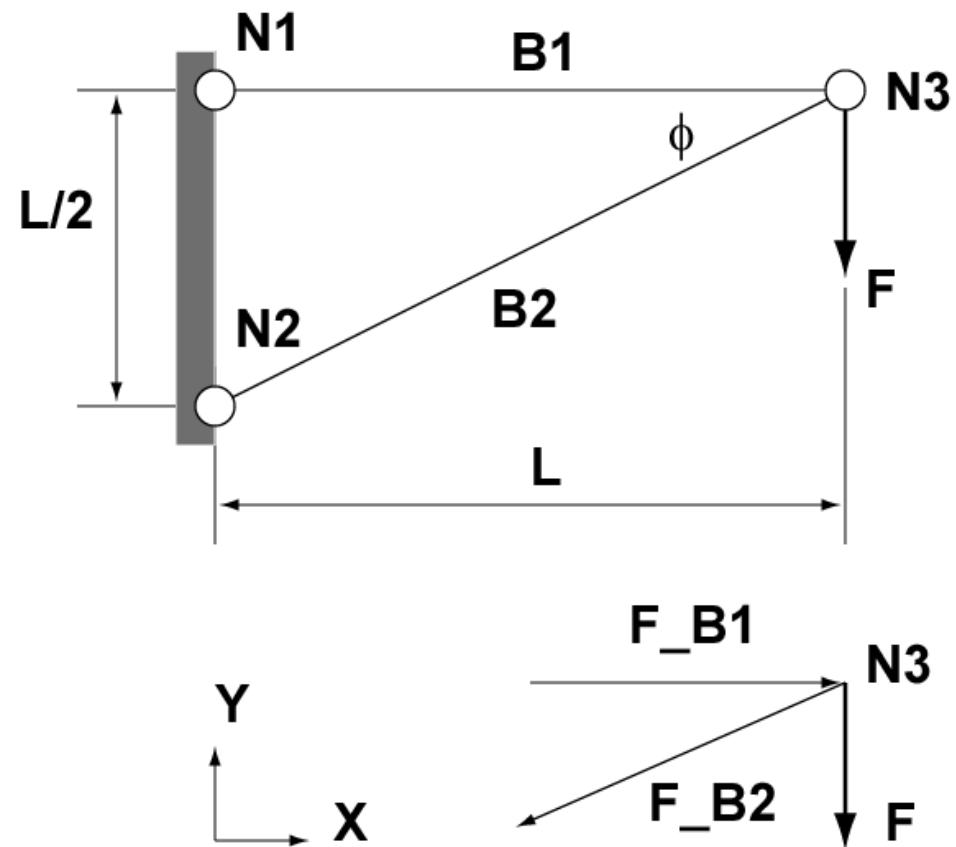
$$F_{b1} = \frac{F}{\sin \phi}$$

N3: x-direction:

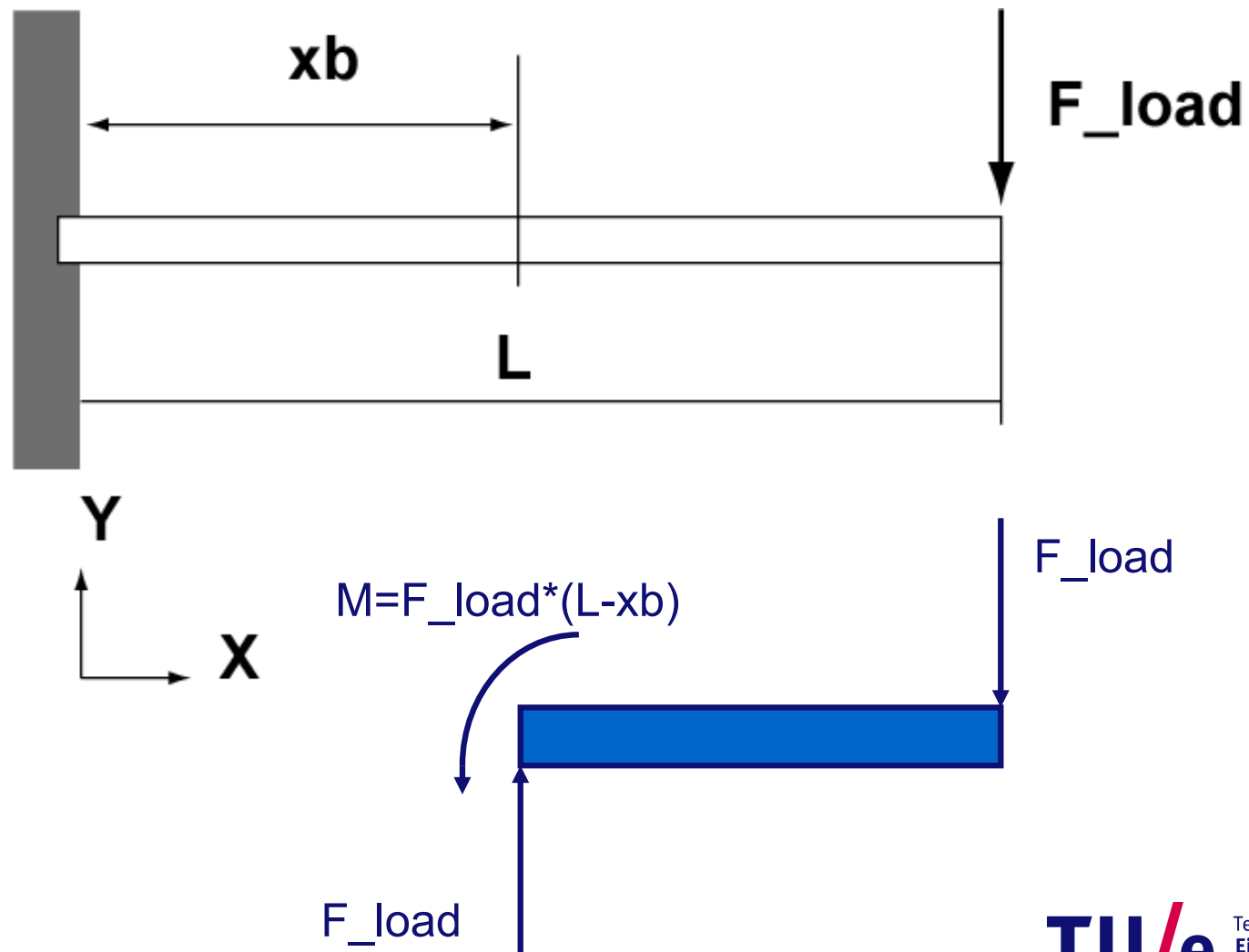
$$-F_{b1} \cos \phi + F_{b2} = 0$$

$$F_{b2} = F_{b1} \cos \phi = \frac{F}{\sin \phi} \cos \phi$$

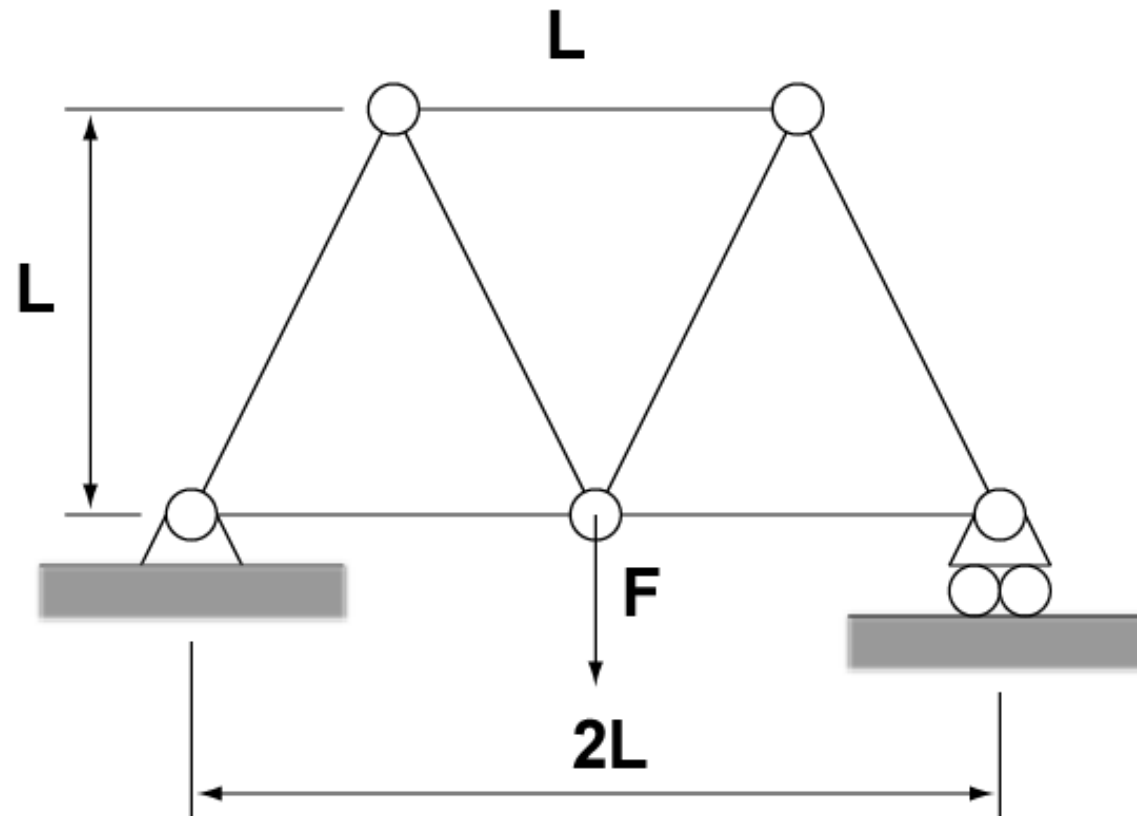
Do it yourself!



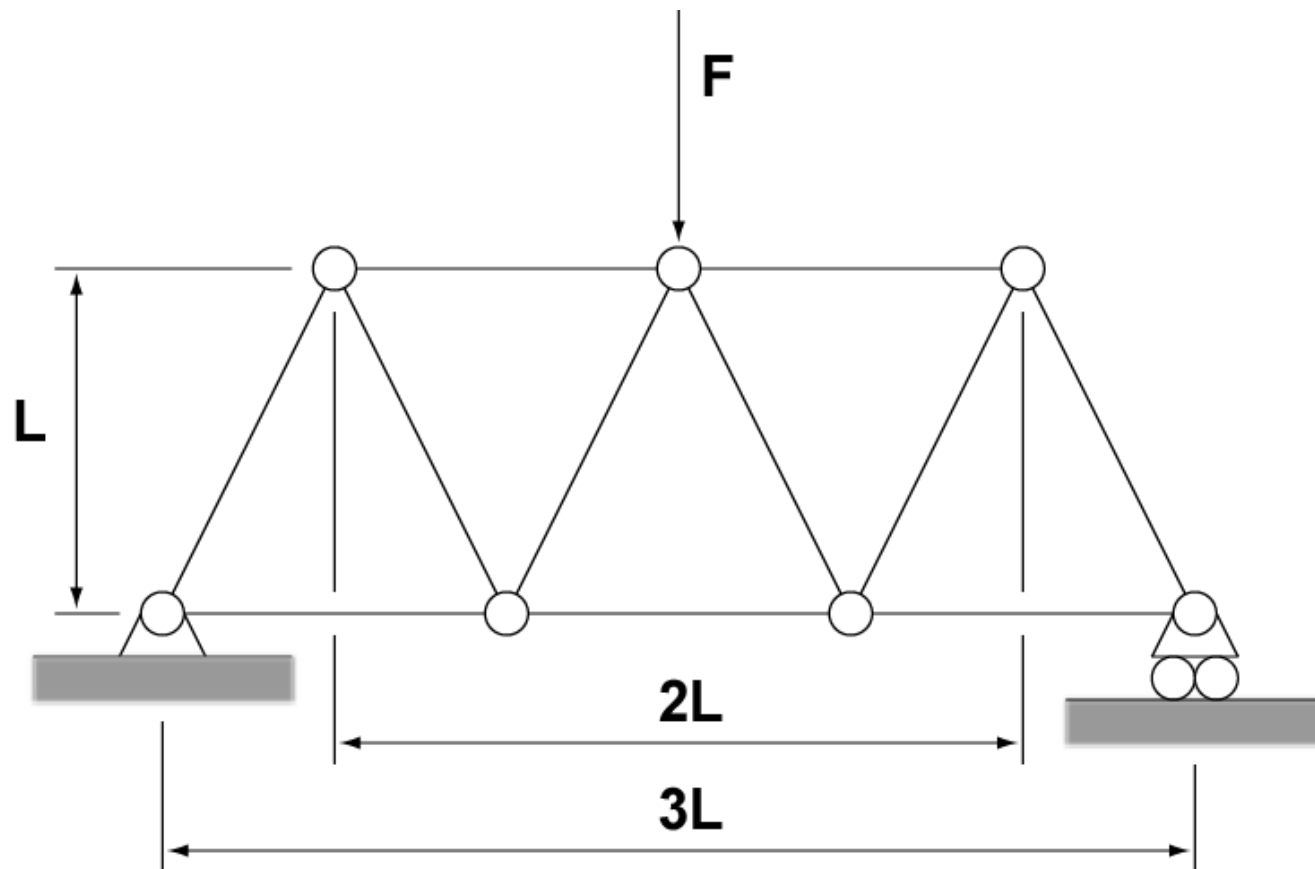
F & M at location xb?



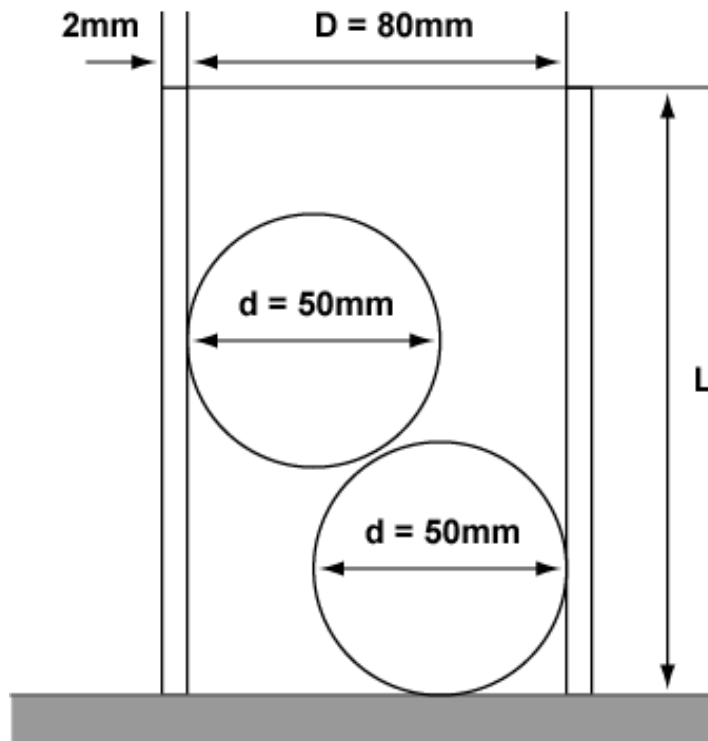
Do it yourself: simple bridge



Do it yourself: simple bridge 2



Do it yourself



- A hollow circular steel cylinder rests on a smooth horizontal surface. The two solid smooth steel spheres are placed inside the cylinder.
- Find the minimum length, L , of the cylinder required if it is not to tip over.

Elasticity



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Definitions:

- **Elasticity:** when removing the load the deformation disappears
- **Plasticity:** the load has caused permanent deformation
- **Mechanical contructions *seldomly* operate in the plastic region!!!**

Elasticity Modulus: E

- **Material property, can be found in literature:**
 - M.F. Ashby, Materials Selection in Mechanical Design
- **Steel: $E = 201\text{-}217 \text{ GPa}$**
- **Aluminum: $E = 68\text{-}82 \text{ GPa}$**
- **Polycarbonate (PC): $E = 2\text{-}2.44 \text{ GPa}$**
- **Polyethyleen (PE): $E = 0.621\text{-}0.896 \text{ GPa}$**

($1 \text{ GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$)

Stresses & strains

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Tensile strain

- **Definition:** $e = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$ in [-]



Stress

- **Stress (tensile / compressive):**

$$\sigma = \frac{F}{A} \text{ in } \frac{N}{m^2}$$

- **Stress concentrations:**

$$K = \frac{\sigma_{\max}}{\sigma_{\text{mean}}}$$

- **Shear stress:**

$$\tau = \frac{F_{\text{shear}}}{A_{\text{shear}}} \text{ in } \frac{N}{m^2}$$

Tensile stress

- **Definition:**

- **Stress:**

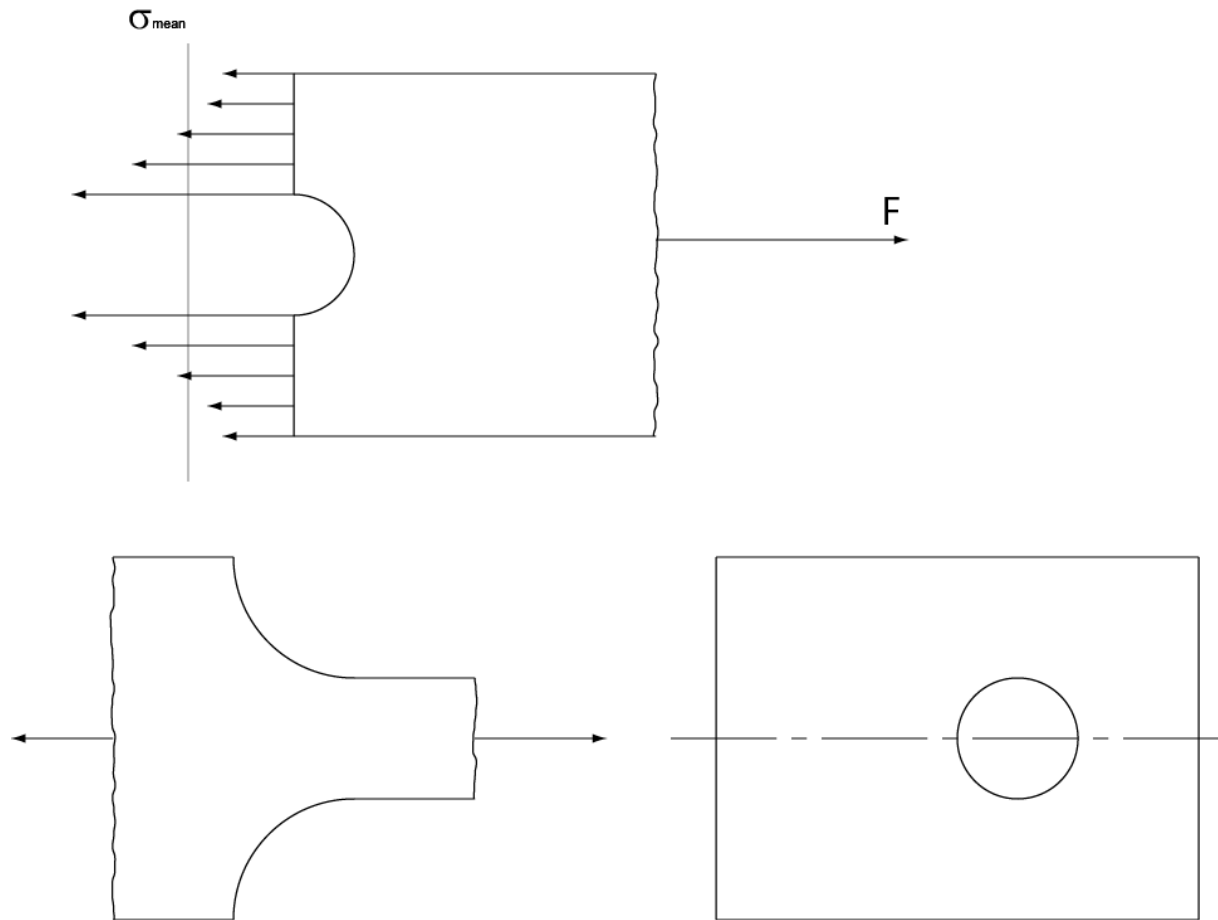
$$\sigma = \frac{F}{A} \text{ in } \frac{N}{m^2} = Pa$$

- **Allowable stresses:**

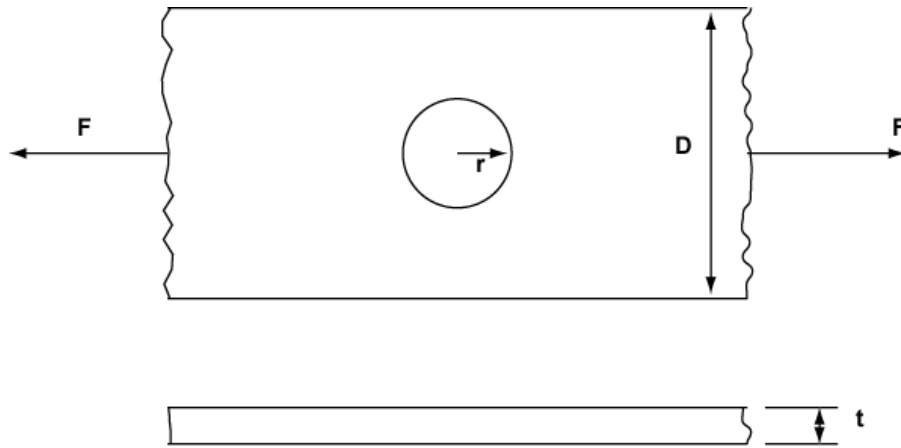
- Safety factor (also called ignorance factor)
 - Allowable material stress σ_y or σ_f (yield/failure)

- **Warning: stress concentrations!**

Stress concentrations!



Stress concentration factors



$$\sigma_{\max} = K_t \cdot \sigma_{\text{nom}}$$

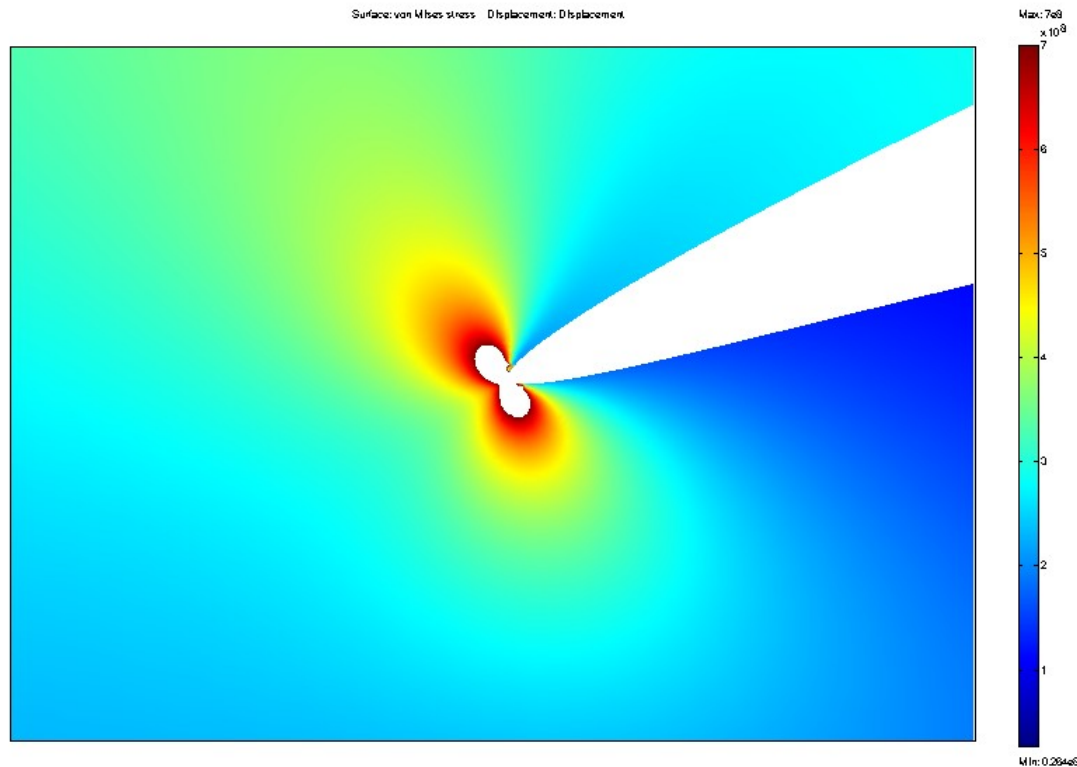
$$\sigma_{\text{nom}} = \frac{F}{t(D - 2r)}$$

$$K_t = 3.00 - 3.13 \cdot \left(\frac{2r}{D} \right) + 3.66 \cdot \left(\frac{2r}{D} \right)^2 - 1.53 \cdot \left(\frac{2r}{D} \right)^3$$

Literature:

- W.C. Young and R. G. Budynas, Roark's formulas for Stress and Strain, Mc. Graw Hill

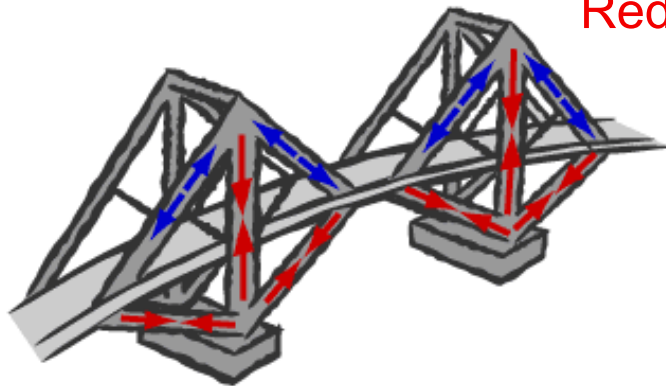
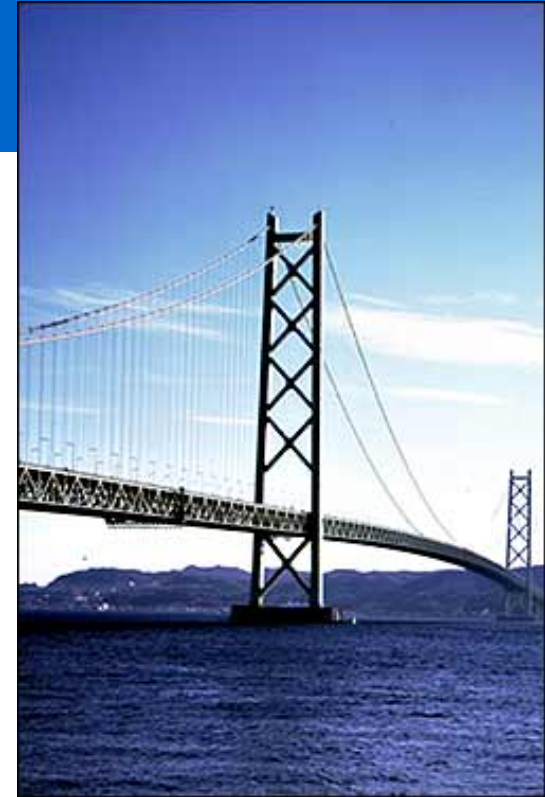
Stressconcentration around a crack



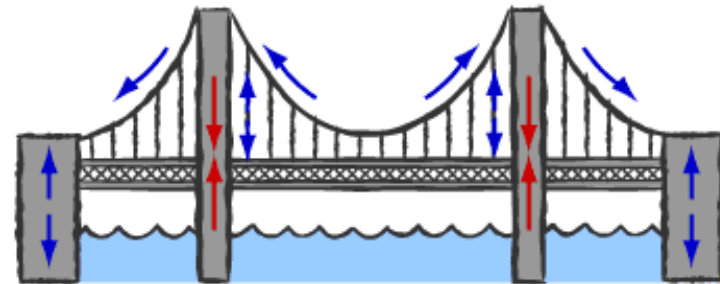
Allowable stresses

- **Material dependent**
- **Safety factor**
 - **Material properties (chemical, physical,...)**
 - **Loading conditions (often poorly known)**
 - **Type of possible failure**
 - **Accuracy of analysis**
 - **Consequences of failure**

Bridge forces/stresses

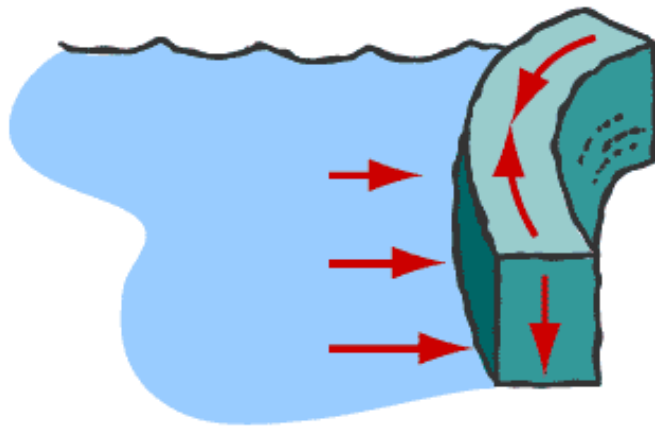


Red: compression stress

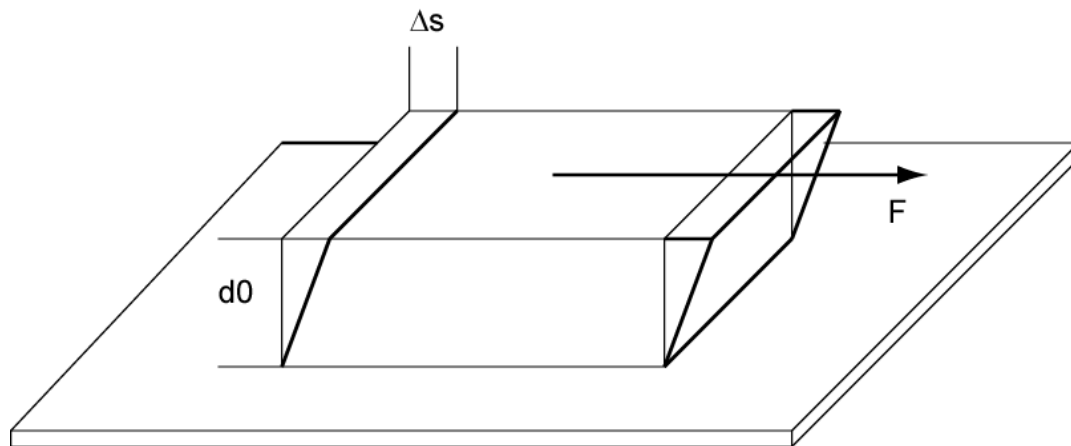


- www.pbs.org/wgbh/buildingbig/bridge/

Dams and domes forces/stresses



Shear stress and shear strain



$$\tau = G \cdot \gamma \text{ in } \frac{N}{m^2}$$

$$\tau = \frac{F}{A} \text{ in } \frac{N}{m^2}$$

$$\gamma = \frac{\Delta s}{d_0} \text{ in } -$$

$$G = \text{shear modulus in } \frac{N}{m^2}$$

Shear modulus G

- Material dependend
- The E modulus and the G modulus are related especially for isotropic materials
- Can be found in for instance
 - M.F. Ashby, Materials Selection in Mechanical Design

$$G = \frac{E}{2(1+\nu)} \text{ in } \frac{N}{m^2}$$

ν = Poisson's ratio

$$\approx \frac{1}{3} \text{ (for isotropic material)}$$

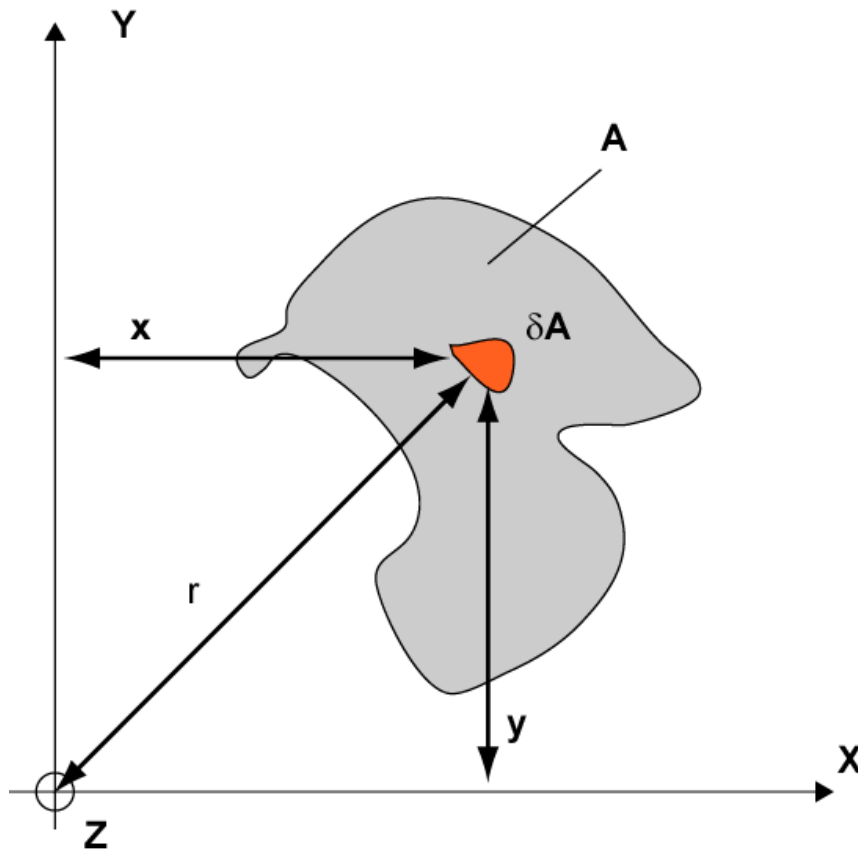
Second moment of Inertia

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Second moment of inertia



$$I_x = \int_A y^2 \cdot dx \cdot dy \text{ in } m^4$$

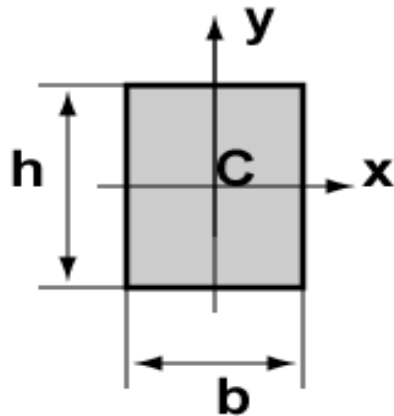
$$I_y = \int_A x^2 \cdot dx \cdot dy \text{ in } m^4$$

$$I_z = \int_A r^2 \cdot dA \text{ in } m^4$$

I of simple shapes

- See also:
 - Pahl & Beitz, Dubbel Taschenbuch fuer den Maschinenbau
 - Fenner R.T., Mechanics of Solids.
 - Leijendeckers et al., Poly-Technisch Zakboek
 - Etc.

I Rectangle



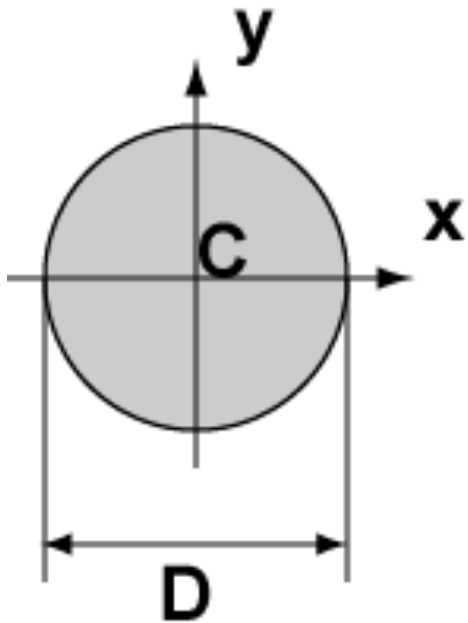
$$(x_C, y_C) = (0, 0)$$

$$I_x = \frac{b \cdot h^3}{12}$$

$$I_y = \frac{h \cdot b^3}{12}$$

$$J_z = \frac{b \cdot h}{12} (b^2 + h^2)$$

I Circle



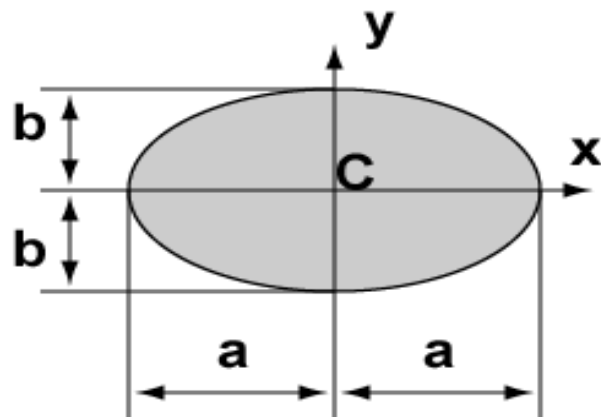
$$(x_C, y_C) = (0, 0)$$

$$I_x = \frac{\pi D^4}{64}$$

$$I_y = \frac{\pi D^4}{64}$$

$$J_z = \frac{\pi D^4}{32}$$

I Ellipse



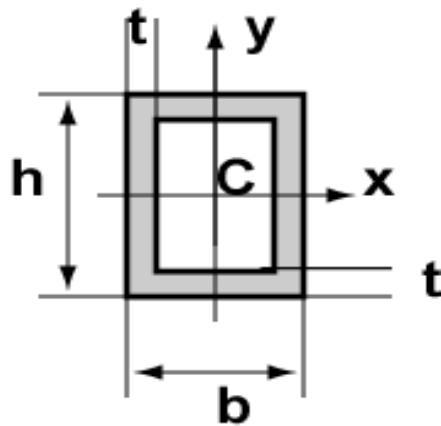
$$(x_C, y_C) = (0, 0)$$

$$I_x = \frac{\pi ab^3}{4}$$

$$I_y = \frac{\pi a^3 b}{4}$$

$$J_z = \frac{\pi ab}{4} (a^2 + b^2)$$

I hollow rectangle



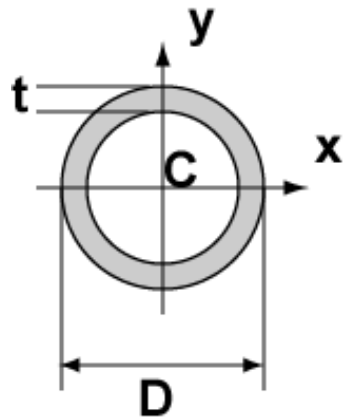
$$(x_C, y_C) = (0, 0)$$

$$I_x = \frac{b \cdot h^3}{12} - \frac{(b - 2t) \cdot (h - 2t)^3}{12}$$

$$I_y = \frac{h \cdot b^3}{12} - \frac{(h - 2t) \cdot (b - 2t)^3}{12}$$

$$J_z = I_x + I_y$$

I hollow circle



$$(x_C, y_C) = (0, 0)$$

$$I_x = \frac{\pi D^4}{64} - \frac{\pi (D - 2t)^4}{64}$$

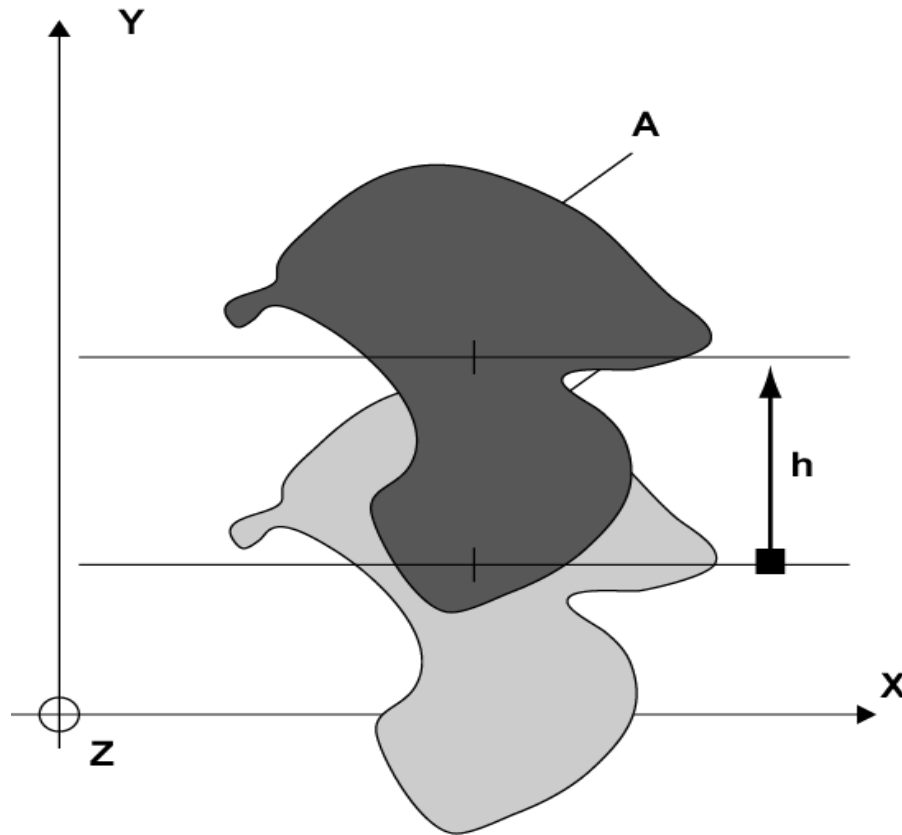
$$I_y = \frac{\pi D^4}{64} - \frac{\pi (D - 2t)^4}{64}$$

$$J_z = \frac{\pi D^4}{32} - \frac{\pi (D - 2t)^4}{32}$$

Parallel axis theorem

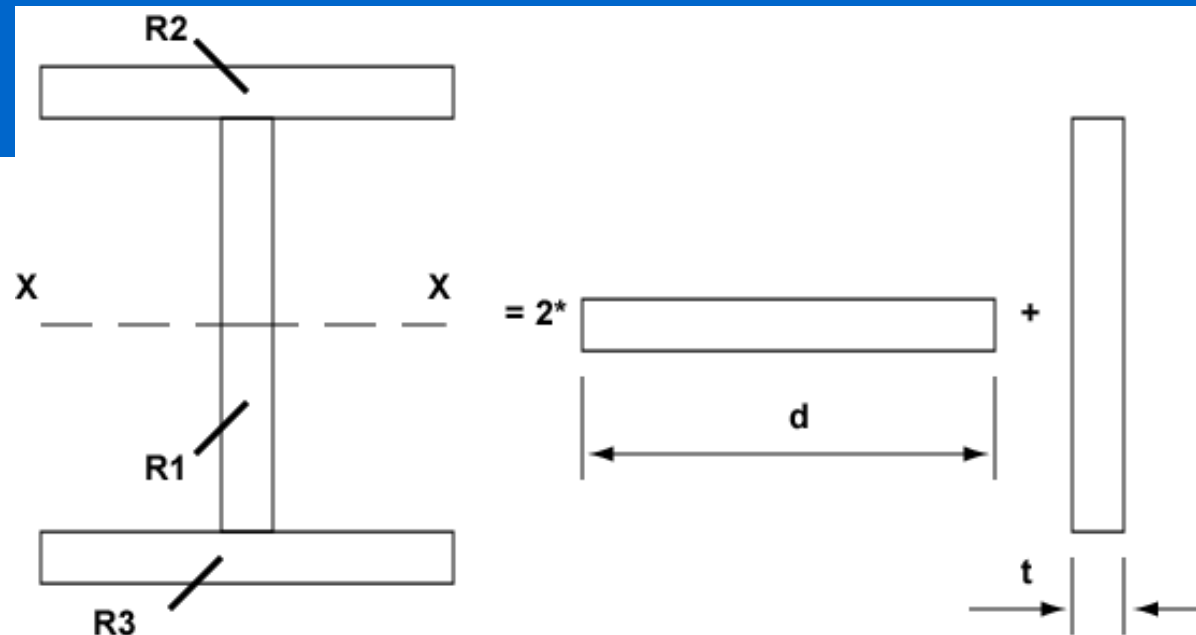
1. Given the second moment of area (moment of inertia) of a region about a particular axis.
2. What is the second moment of area (moment of inertia) around an axis parallel to the above mentioned axis?

Parallel axis theorem



$$I' = I + A \cdot h^2$$

Example I profile:



$$R1: I_x = \frac{bh^3}{12} = \frac{td^3}{12}$$

$$R2: I_x = \frac{bh^3}{12} + r_y^2 \cdot A = \frac{dt^3}{12} + \left(\frac{d}{2} + \frac{t}{2} \right)^2 \cdot d \cdot t$$

$$R3: I_x = \frac{bh^3}{12} + r_y^2 \cdot A = \frac{dt^3}{12} + \left(-\frac{d}{2} - \frac{t}{2} \right)^2 \cdot d \cdot t$$

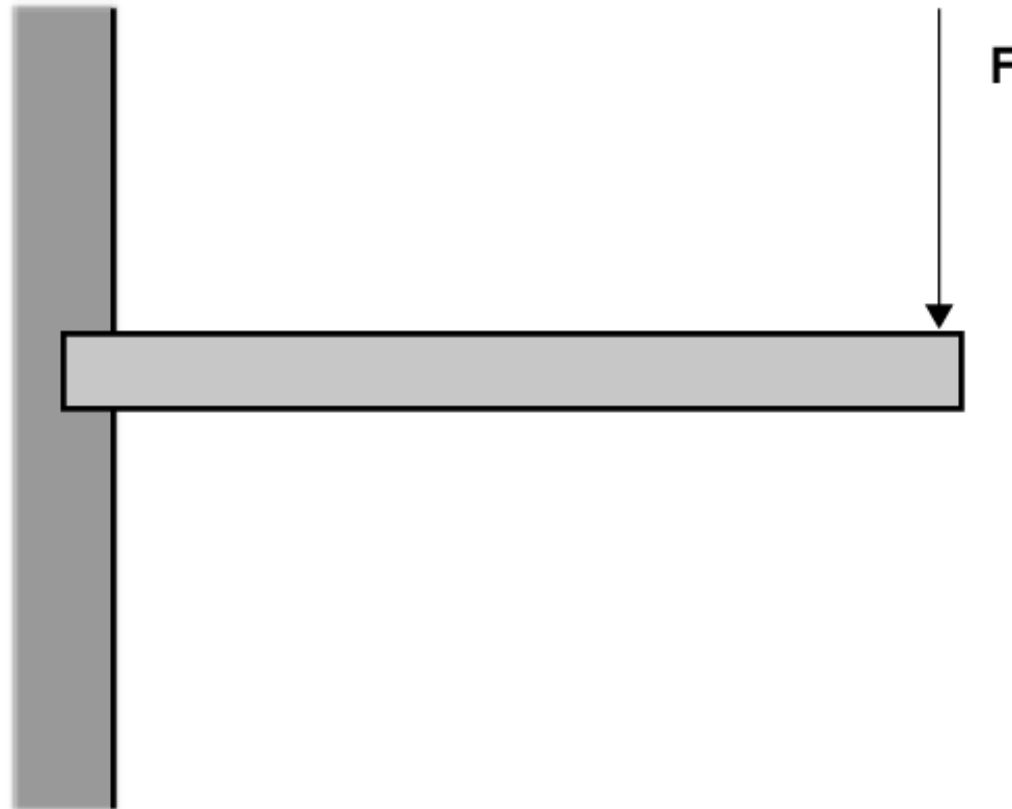
$$I_x = \frac{td^3}{12} + \frac{dt^3}{6} + 2 \cdot \left(\frac{d}{2} + \frac{t}{2} \right)^2 \cdot d \cdot t$$

Loading types & diagrams

Loading types

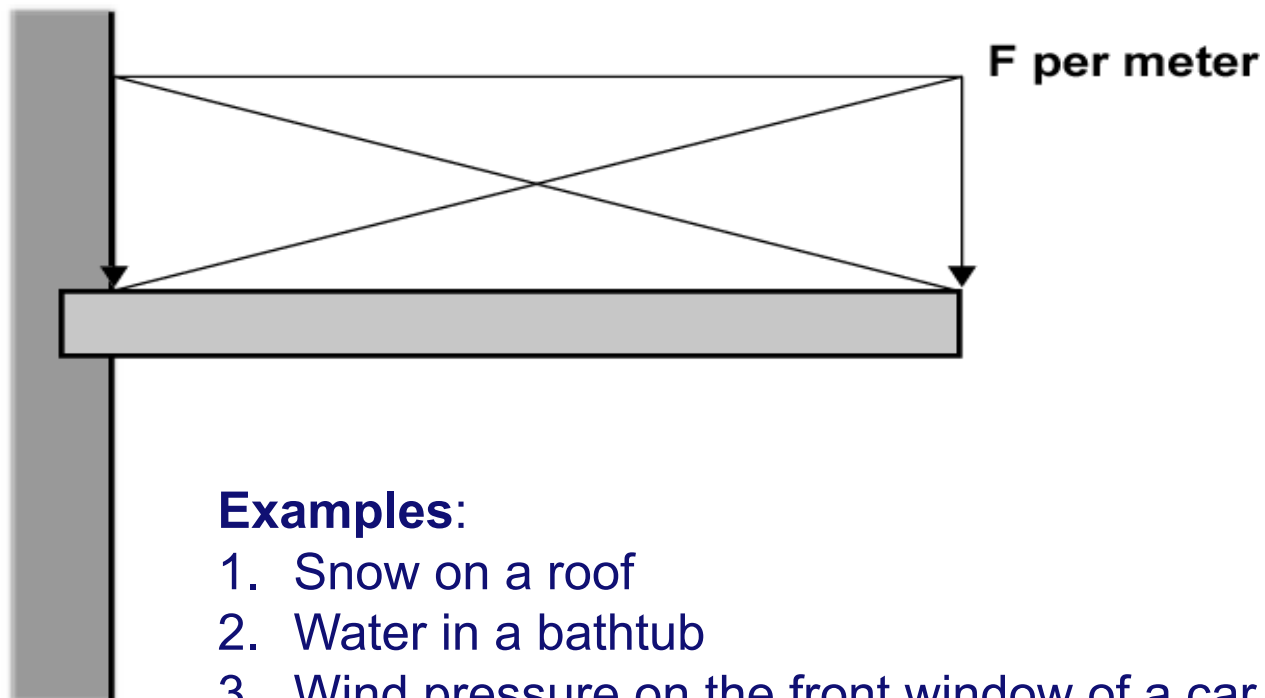
- Point force or point torque
- Distributed force
- Examples

Point force load



Example: weight hanging on a beam

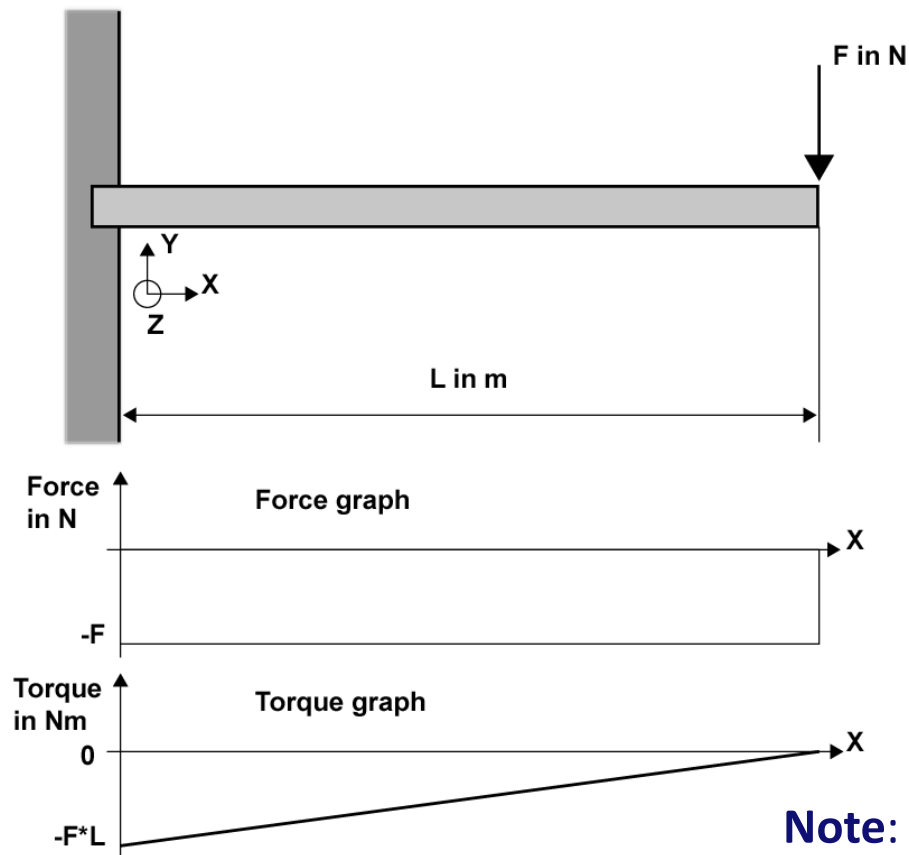
Distributed load



Examples:

1. Snow on a roof
2. Water in a bathtub
3. Wind pressure on the front window of a car while driving

Loading diagrams



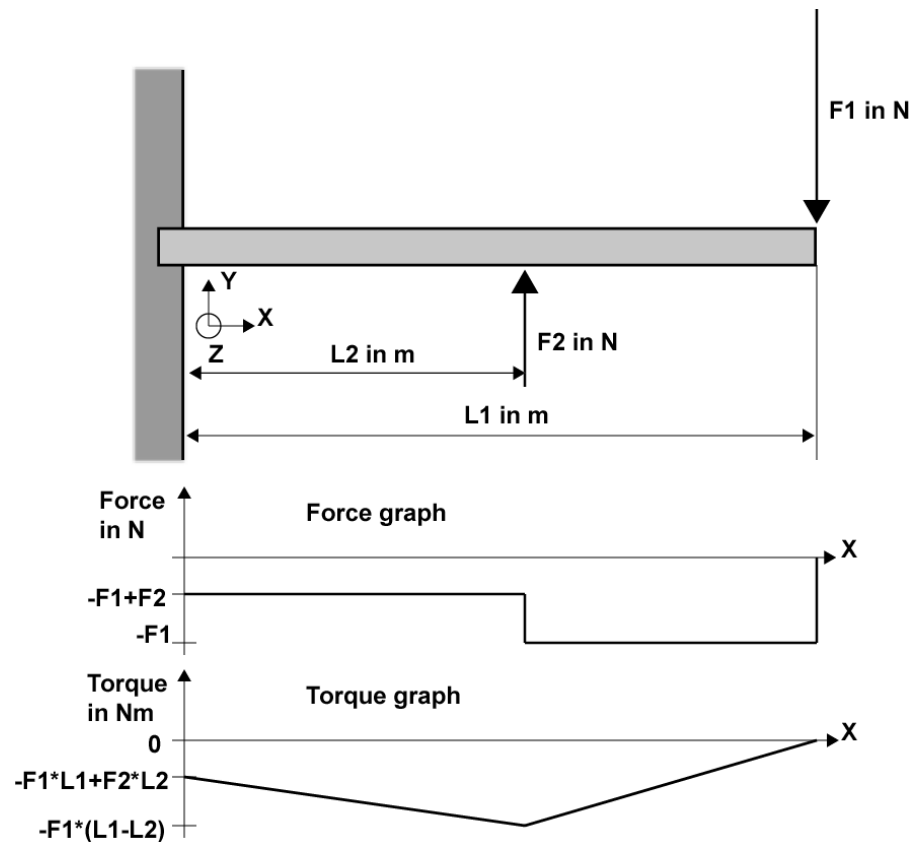
$$0 < x < L$$

$$\Rightarrow F_{load} = -F$$

$$\Rightarrow M_{load} = -F \cdot (L - x)$$

Note: reaction forces and torques of the beam are the same size but opposite in sign!

Loading diagram



$$L_2 < x < L_1$$

$$\Rightarrow F_{load} = -F_1$$

$$\Rightarrow M_{load} = -F_1 \cdot (L_1 - x)$$

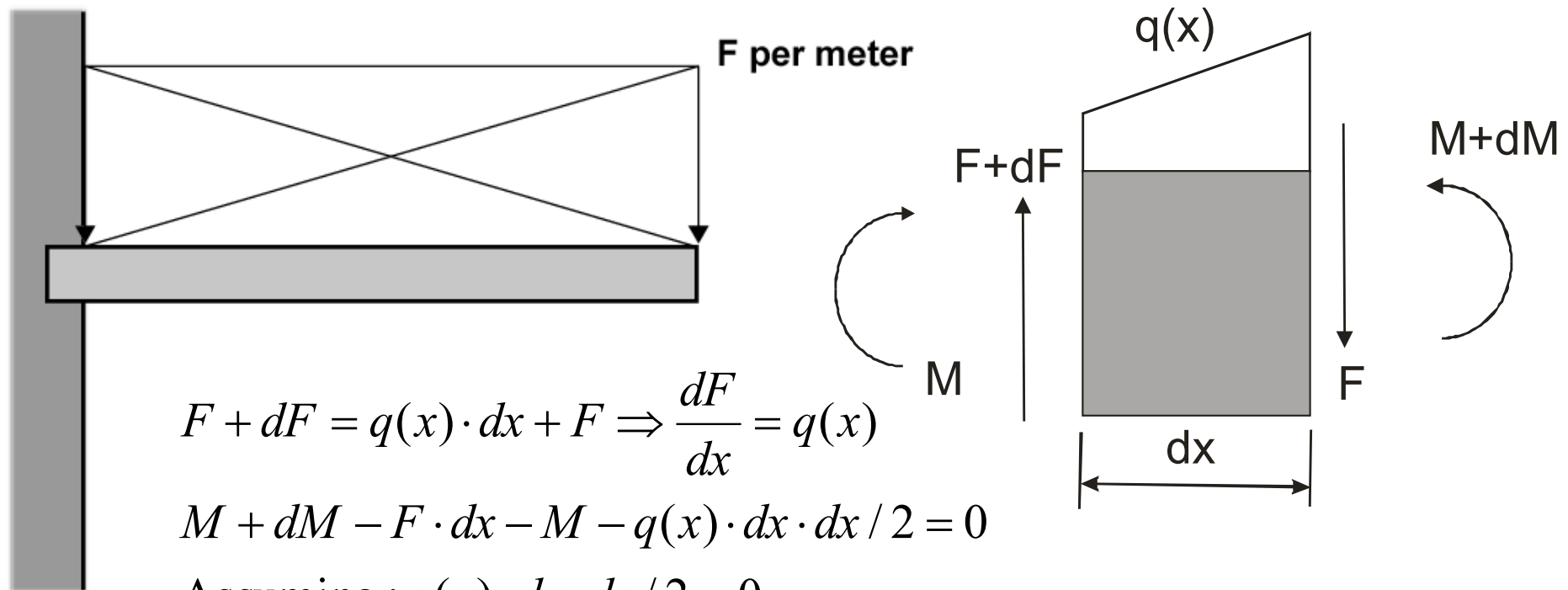
$$0 < x < L_2$$

$$\Rightarrow F_{load} = -F_1 + F_2$$

$$\Rightarrow M_{load} = -F_1 \cdot (L_1 - x)$$

$$+ F_2 \cdot (L_2 - x)$$

Loading diagrams



$$F + dF = q(x) \cdot dx + F \Rightarrow \frac{dF}{dx} = q(x)$$

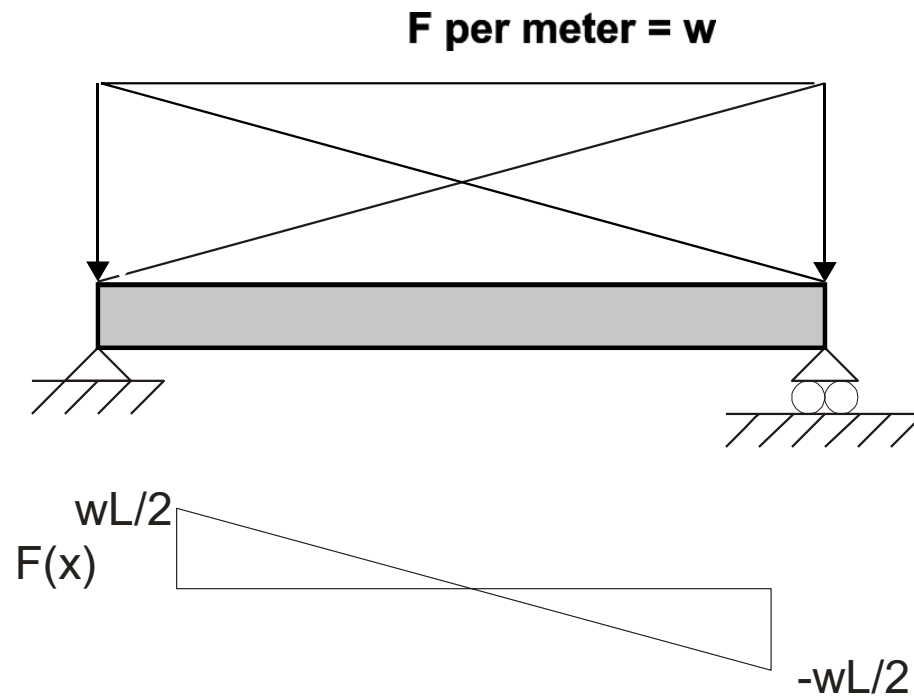
$$M + dM - F \cdot dx - M - q(x) \cdot dx \cdot dx / 2 = 0$$

$$\text{Assuming : } q(x) \cdot dx \cdot dx / 2 \approx 0$$

$$\frac{dM}{dx} = F \text{ or : } \frac{d^2 M}{dx^2} = q(x)$$

With the appropriate boundary conditions the force and the moment can be determined at each point.

Example:



$$q(x) = -w$$

$$F(x) = \int_0^x q(x) dx = -wx + C_1$$

$$F(0) = wL/2 \Rightarrow C_1 = wL/2$$

$$F(x) = -wx + wL/2$$

$$M(x) = \int_0^x F(x) dx$$

$$M(x) = -wx^2/2 + wxL/2 + C_2$$

$$M(0) = 0 \Rightarrow C_2 = 0$$

$$= wx(L-x)/2$$



Loading reactions & optimal material use

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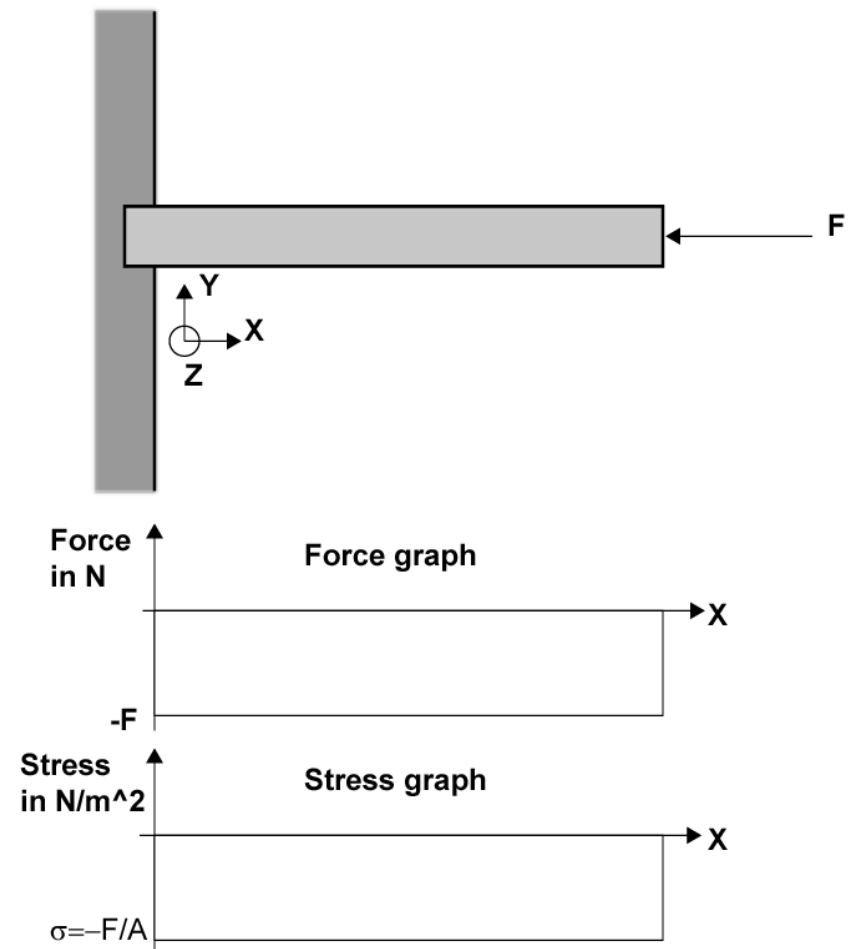
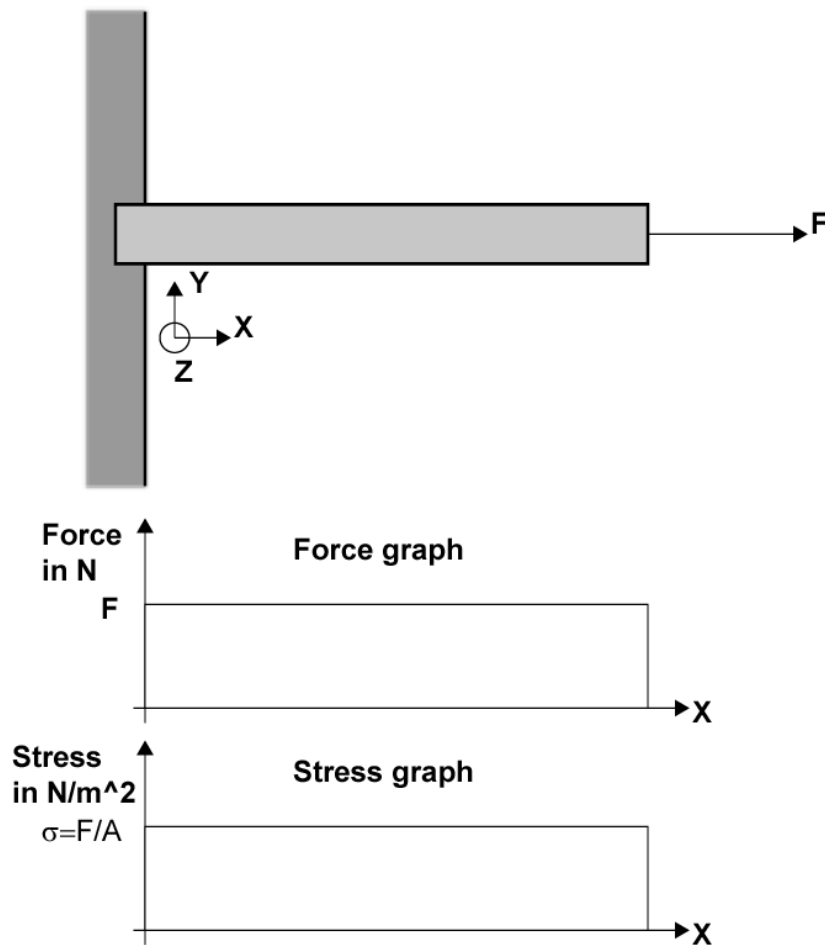
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Loading reactions in simple beams

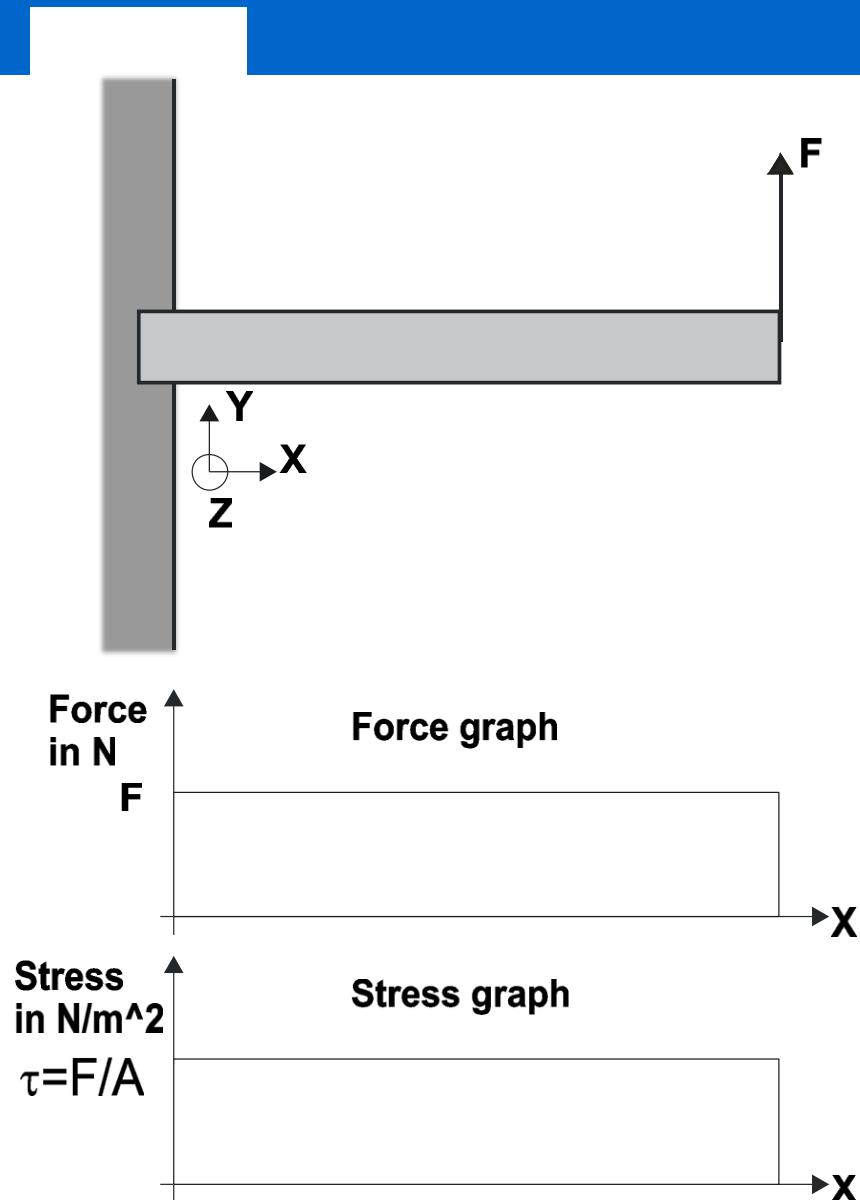
- Tension
- Compression
 - Buckling!
- Shear
- Bending
- Torsion

- Examples of each

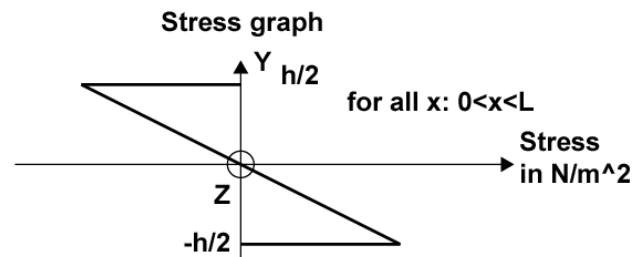
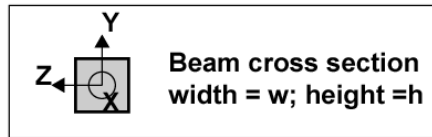
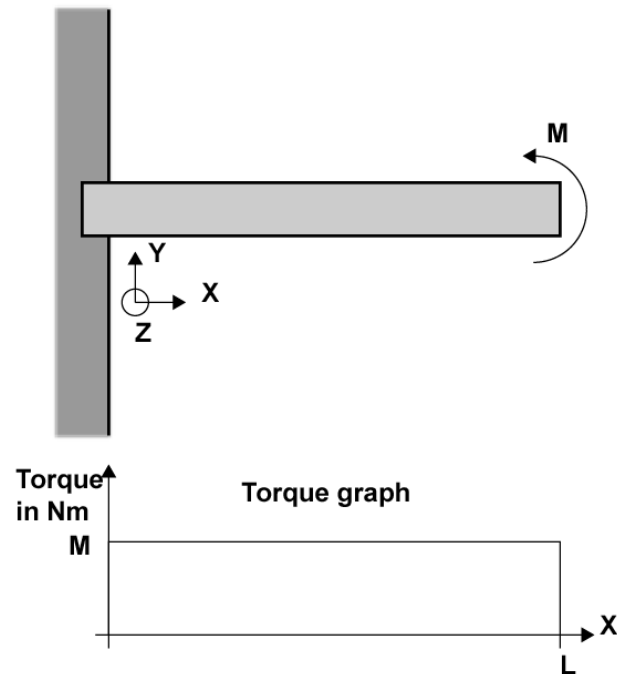
Tension and compression



Shear



Bending



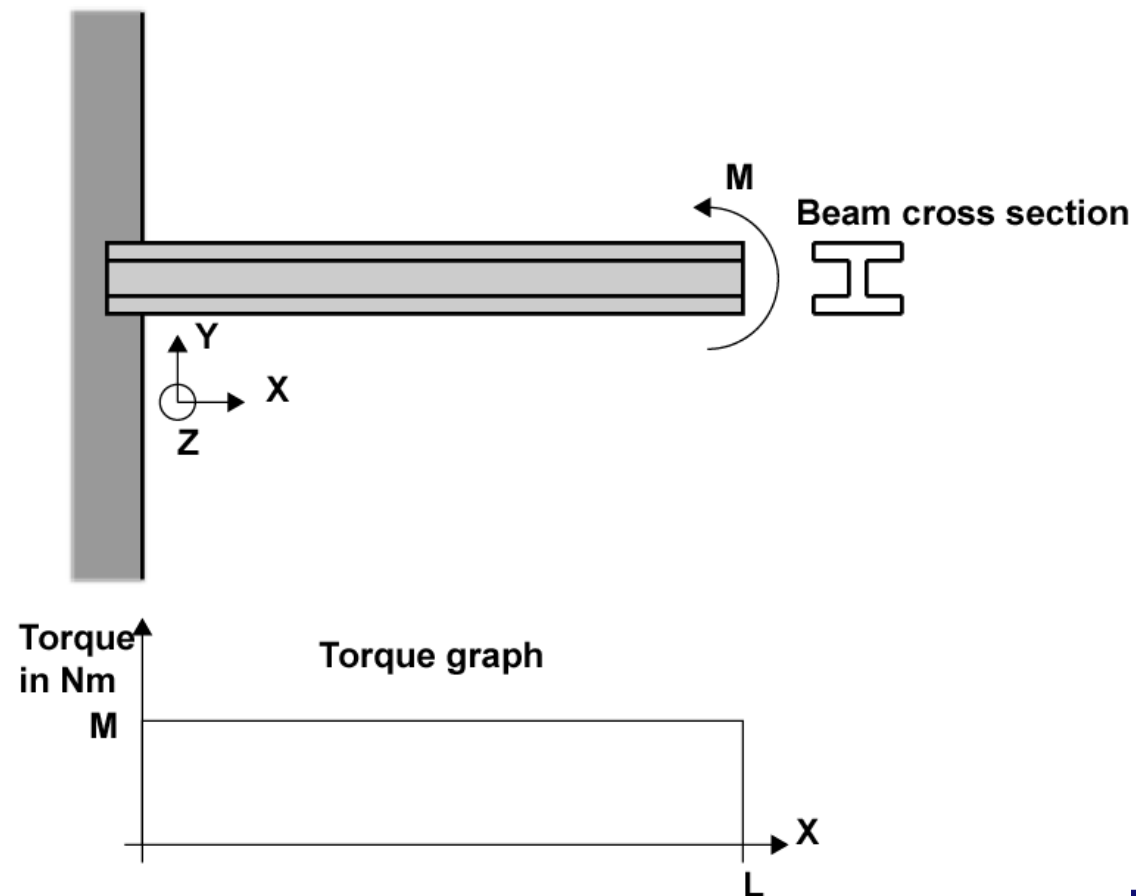
$$I_z = \frac{w \cdot h^3}{12}$$

$$\sigma_x = -\frac{y \cdot M}{I_z}$$

Bending optimal material use:

- Highest stresses at the outer fibers
- Thus material used most effectively at the outer fibers!

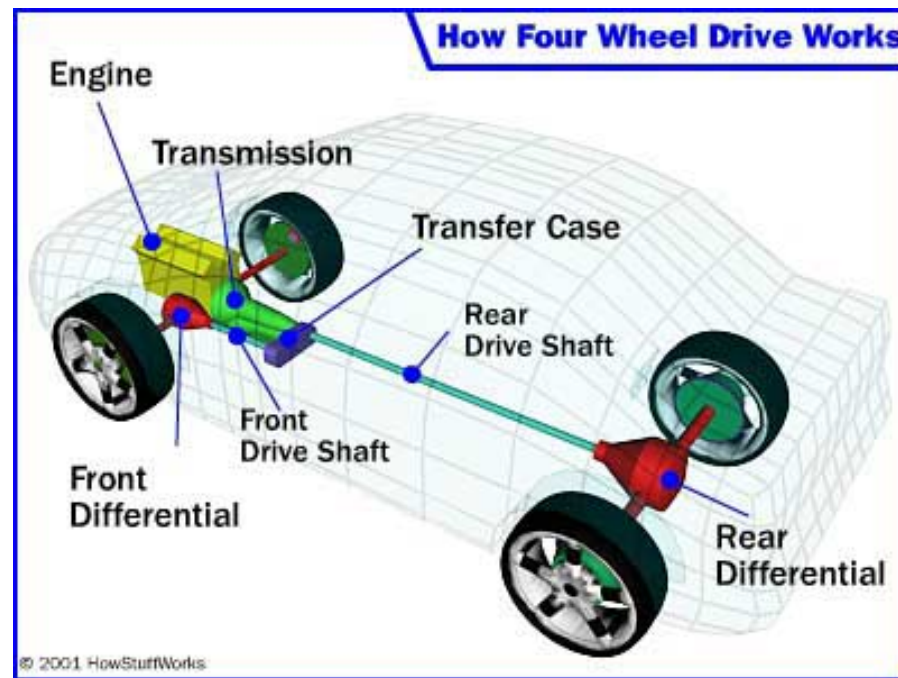
Optimal material use for bending

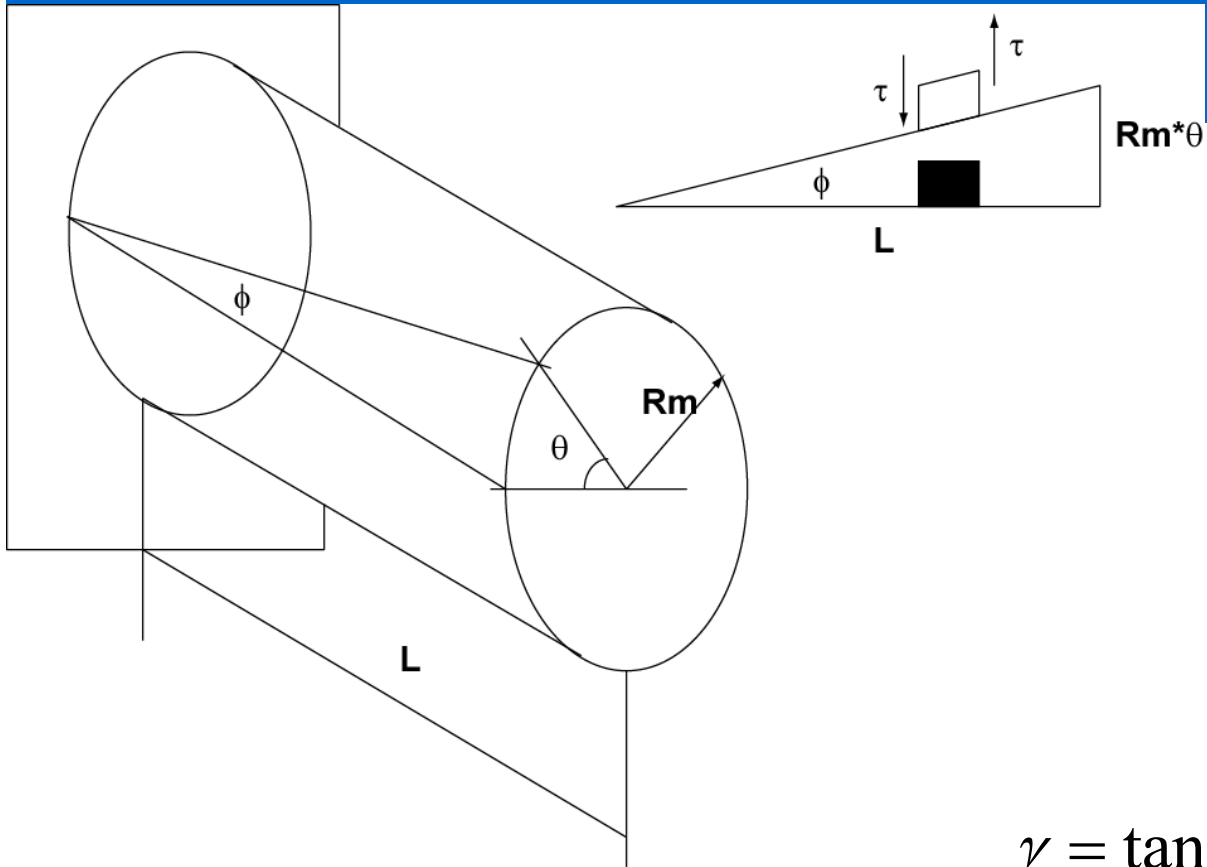


Torsion



- Stress shape
- Optimal material use

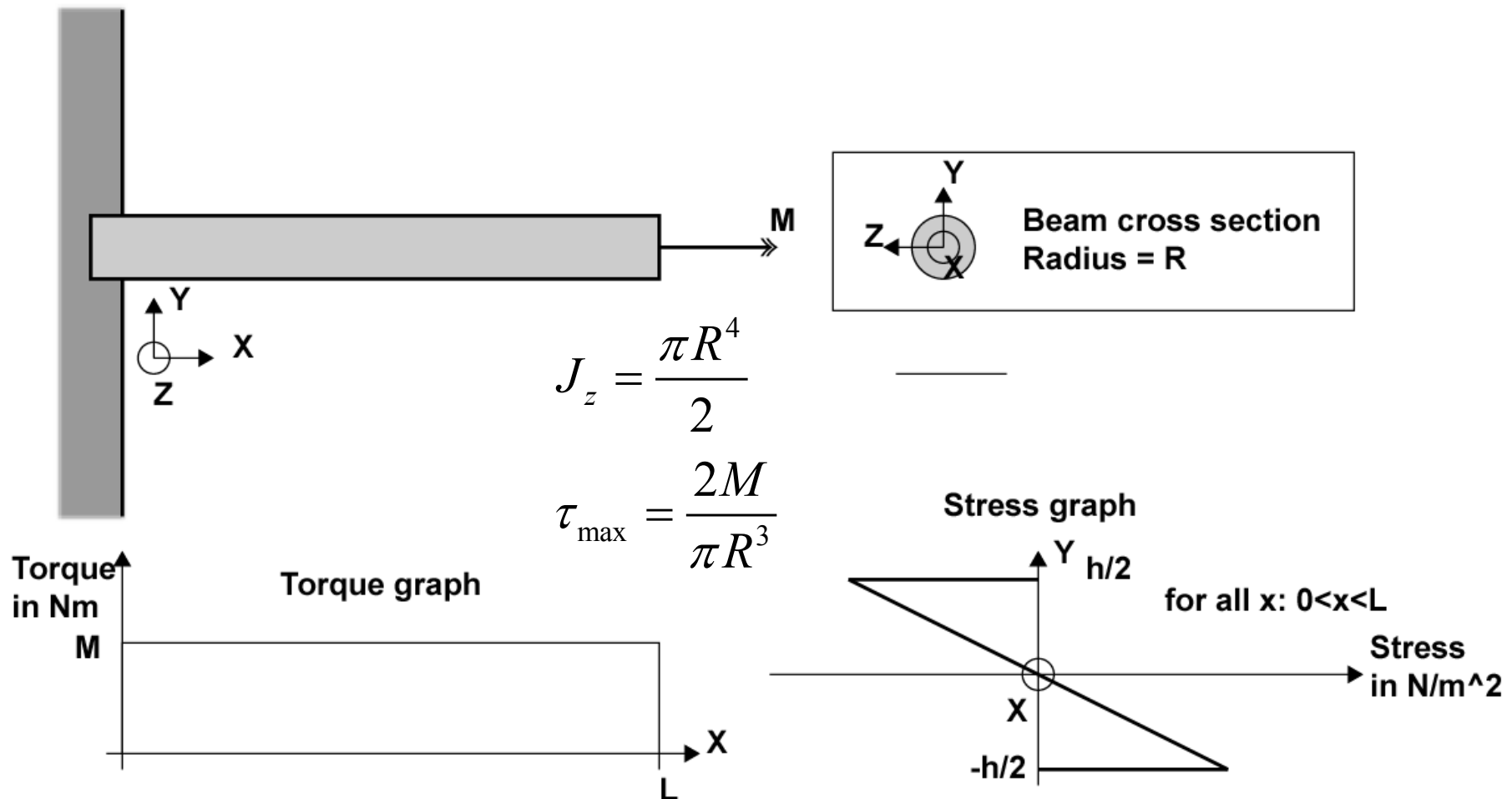




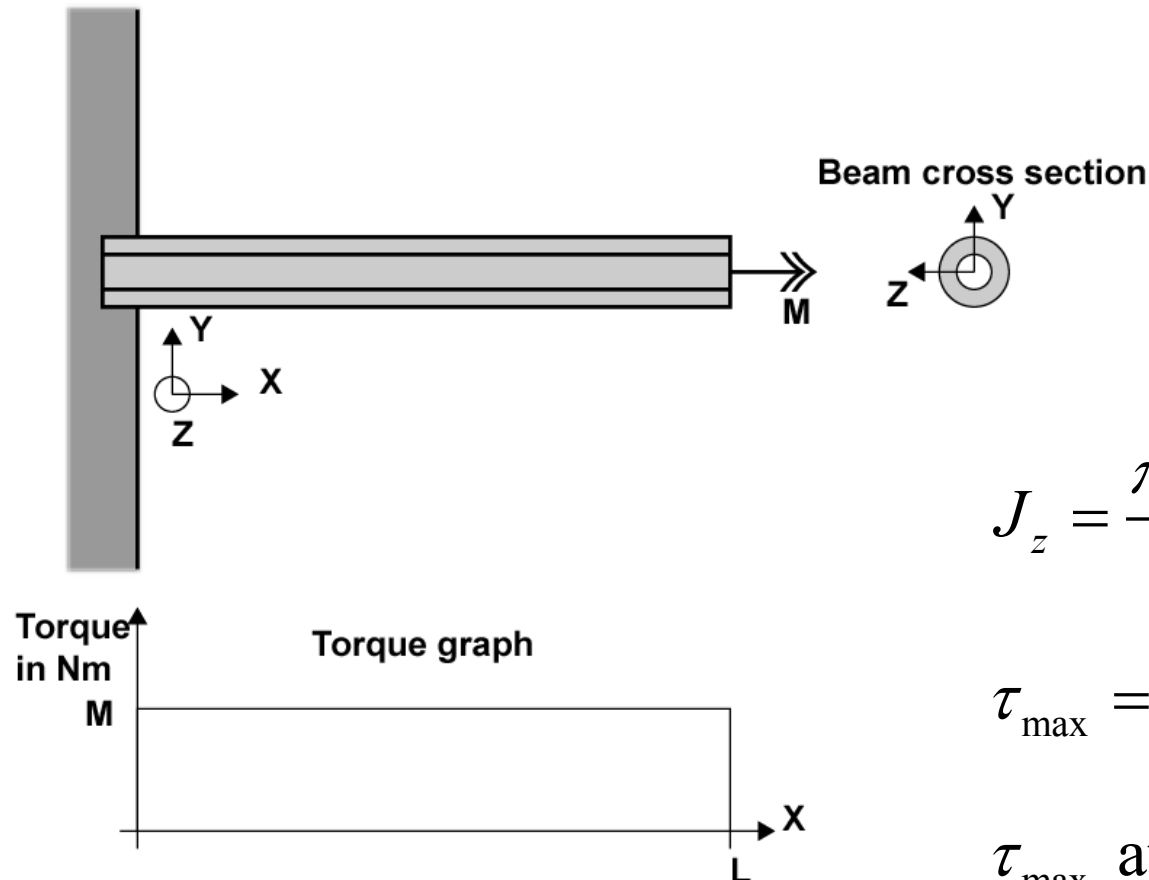
$$\gamma = \tan \phi = \frac{R_m \cdot \theta}{L}$$

$$\tau = G \cdot \gamma = G \cdot \frac{R_m \cdot \theta}{L}$$

Torsion



Torsion, optimal material use



$$J_z = \frac{\pi(R_{outer} - R_{inner})^4}{2}$$

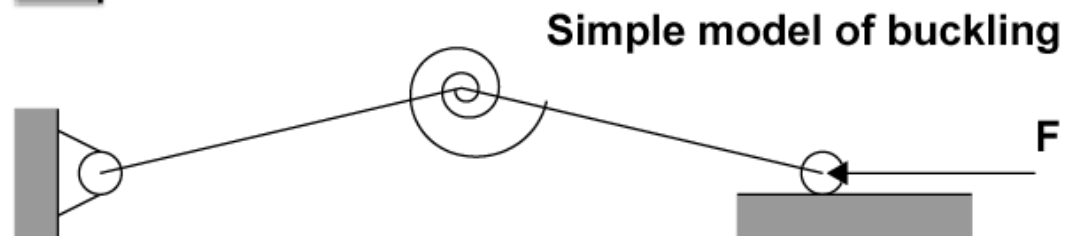
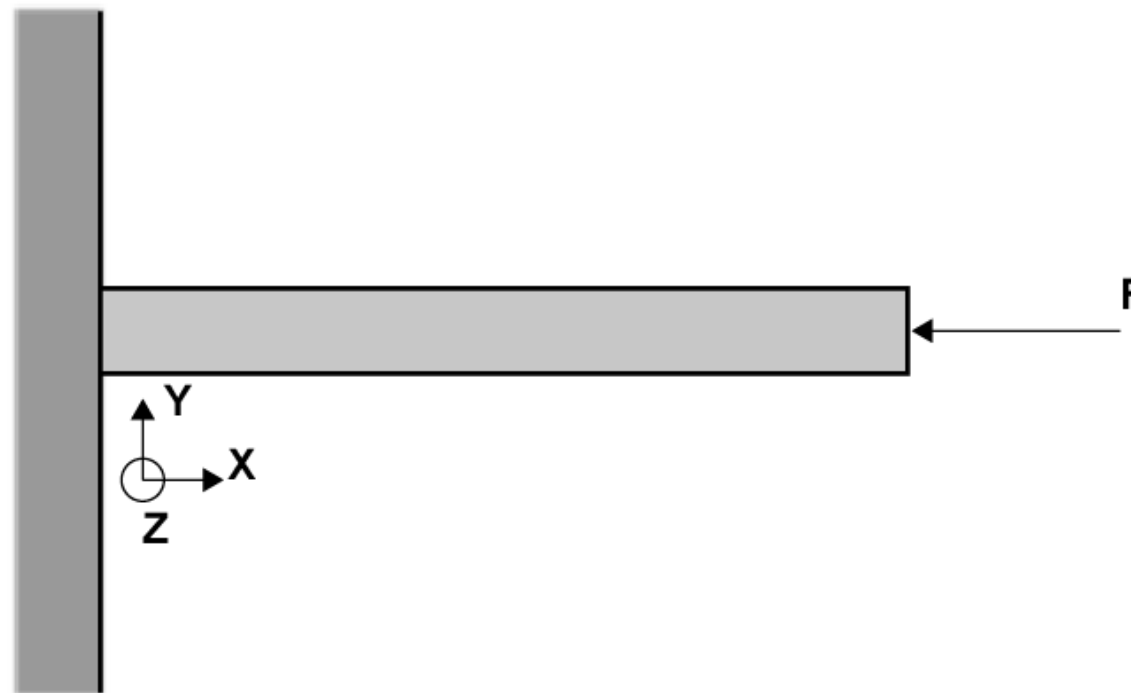
$$\tau_{max} = \frac{2MR_{outer}}{\pi(R_{outer}^4 - R_{inner}^4)}$$

τ_{max} at outer border

Buckling

- When loading a beam by compression at a certain load the beam bends sideways
- and very often *catastrophically!!*
- Buckling is unstable!
- Try it yourself:
 - a coffee stirrer compression loaded buckles

Buckling simple model



Buckling optimal material use

$$F_{critical} = \frac{K \cdot \pi^2 \cdot E \cdot I}{L^2} \text{ in } N$$

K = constant depending on boundary conditions in -

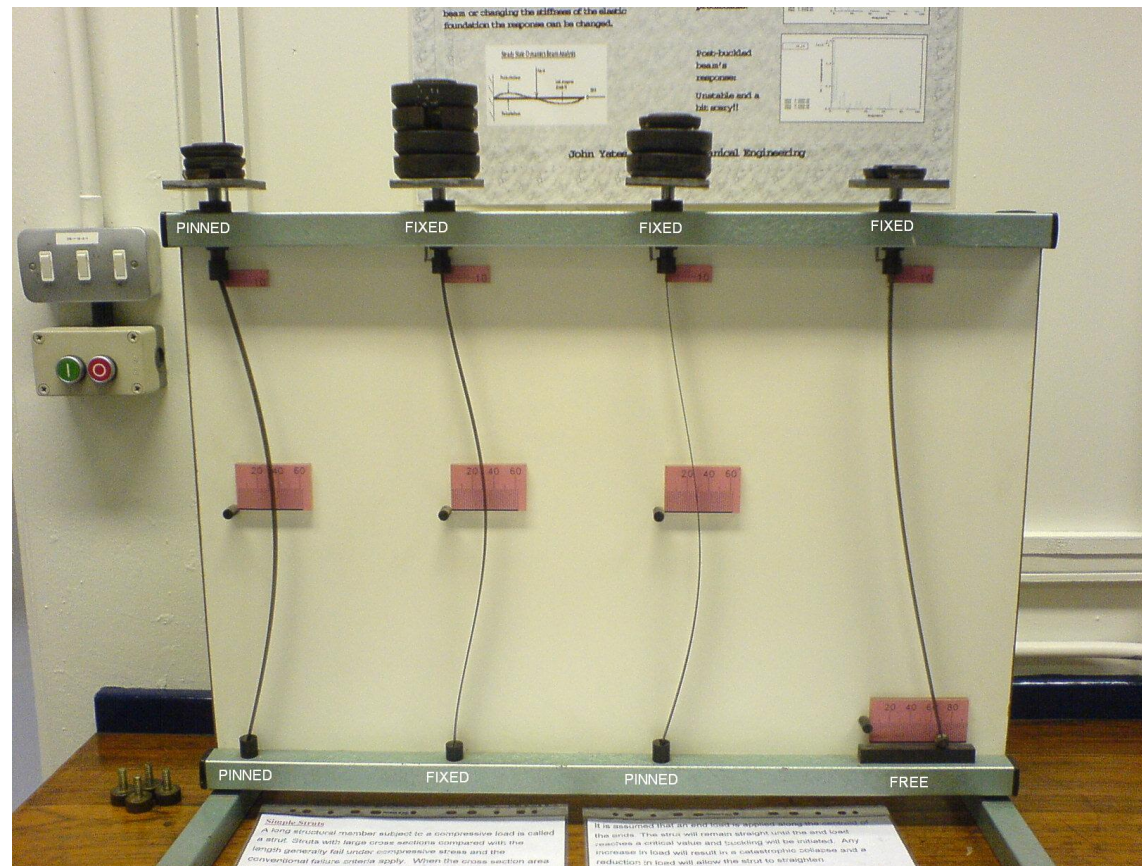
E = modulus of elasticity in $\frac{N}{m^2}$

I = moment of inertia in m^4

L = beam length in m

- **Prevent catastrophical bending**
 - Beam length short (example: Cranes)
 - Outer fibre more material, thus I bending large, the same as for bending!

Buckling cases



$K=1$

$K=4$

$K \approx 2$

$K=1/4$

Buckling and cranes



Example: determine critical load

- Rectangular hollow beam
 - width w ($=20\text{mm}$),
 - height h ($=15\text{mm}$),
 - length L ($=1\text{m}$),
 - thickness t ($=2\text{mm}$)
- Material beam is Steel
- What is the critical load F ?



Deformations due to loading

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Where innovation starts

Deformations

- Materials are not rigid but elastic.
- Thus, they deform under loading
- How much?
 - Depends on the loading
 - Material
 - Geometrical shape,
 - Moment of inertia I

Elongation

- Due to tension/compression
- How to reduce?
- Examples

Relation between tensile stress and strain

$$\sigma = E \cdot \varepsilon$$

$$\sigma = \text{tensile stress in } \frac{N}{m^2}$$

$$E = \text{Young's modulus (modulus of elasticity) in } \frac{N}{m^2}$$

$$\varepsilon = \text{strain in the elastic region in } \frac{m}{m} = -$$

Elongation under tensile load

$$\varepsilon = \frac{\sigma}{E} \text{ in } -$$

$$\sigma = \frac{F}{A} \text{ in } \frac{N}{m^2}$$

$$\Delta l = L_0 \cdot \varepsilon = \frac{L_0 \cdot F}{E \cdot A} \text{ in } m$$

Example: elongation

- Beam round 10mm, length $l = 1\text{m}$
- Material: Steel, $E = 210\text{ GPa}$
- Force: 10 kN

$$\sigma = \frac{F}{A} = \frac{F}{\frac{\pi}{4}d^2}$$

$$\varepsilon = \frac{\sigma}{E_{\text{Steel}}} = \frac{F}{\frac{\pi}{4}d^2 \cdot E_{\text{Steel}}}$$

$$\Delta l = l \cdot \varepsilon = \frac{F \cdot l}{\frac{\pi}{4}d^2 \cdot E_{\text{Steel}}} = \frac{1 \cdot 10^4}{\frac{\pi}{4} \cdot 0.01^2 \cdot 210 \cdot 10^9} = 0.0006\text{m}$$

Compression stress

- The same as tensile stress but do not forget buckling!

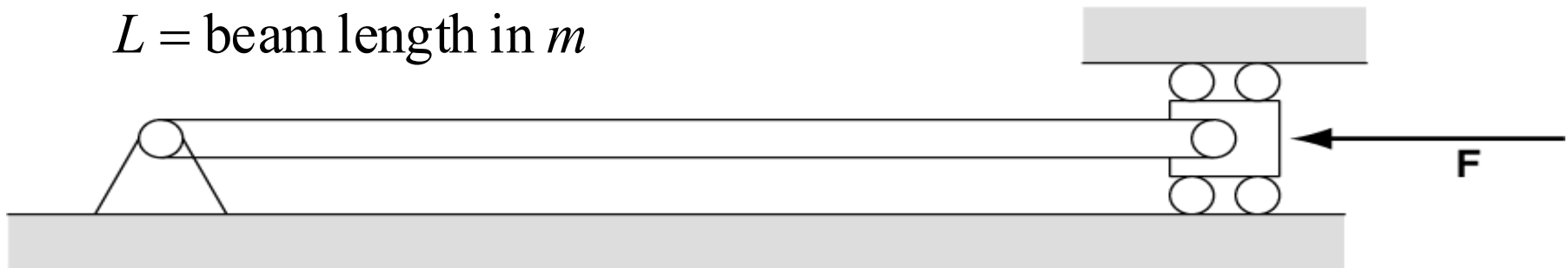
$$F_{critical} = \frac{K \cdot \pi^2 \cdot E \cdot I}{L^2} \text{ in } N$$

K = constant depending on boundary conditions in -

E = modulus of elasticity in $\frac{N}{m^2}$

I = moment of inertia in m^4

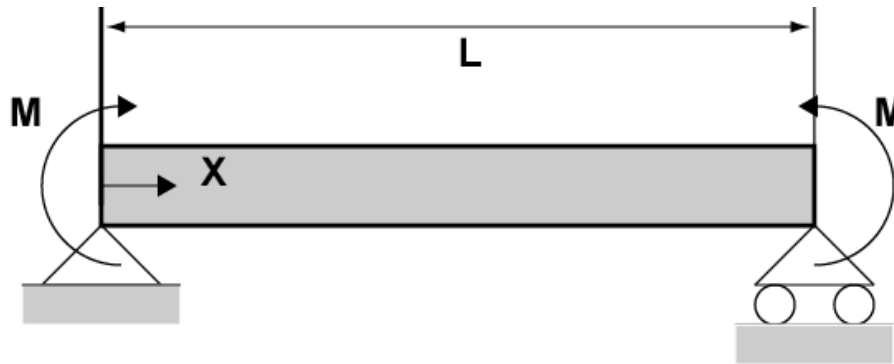
L = beam length in m



Deflection

- Due to bending
- How to reduce?
- Examples

Bending of beams deflection



$$\varepsilon_x = \frac{(R(x) - y) \cdot \delta\theta - R(x) \cdot \delta\theta}{R(x) \cdot \delta\theta} = -\frac{y}{R(x)}$$

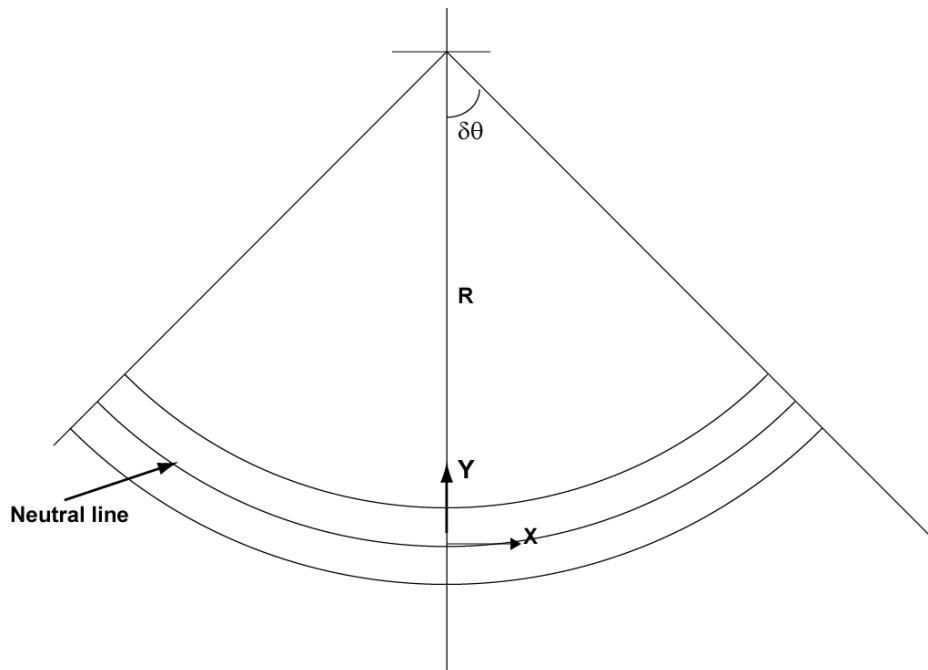
$$\sigma_x = E \cdot \varepsilon_x = -\frac{y \cdot E}{R(x)}$$

$$M(x) = \int -y \cdot \sigma_x \cdot \delta A = \int_{-h/2}^{h/2} -\sigma_x \cdot y \cdot b(y) \cdot dy$$

$$M(x) = \frac{E}{R(x)} \int_{-h/2}^{h/2} y^2 \cdot b(y) \cdot dy$$

$$I_z = \int_{-h/2}^{h/2} y^2 \cdot b(y) \cdot dy$$

$$M(x) = \frac{E \cdot I_z}{R(x)}$$



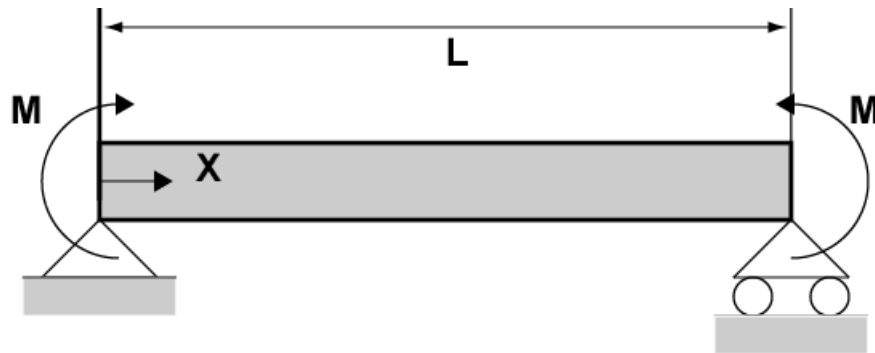
Deflection v base equation:

$$M(x) = \frac{E}{R(x)} I_z(x)$$

$$\frac{1}{R(x)} = \frac{d^2 v}{dx^2} \text{ Radius of curvature definition}$$

$$\Rightarrow \frac{d^2 v}{dx^2} = \frac{M(x)}{E \cdot I_z(x)}$$

Application of the theory



$$E \cdot I \cdot \frac{d^2 v}{dx^2} = M$$

$$E \cdot I \cdot \frac{dv}{dx} = M \cdot x + A$$

$$E \cdot I \cdot v = \frac{1}{2} M \cdot x^2 + A \cdot x + B$$

Add boundary conditions

$$x = 0 \Rightarrow v = 0$$

$$x = L \Rightarrow v = 0$$

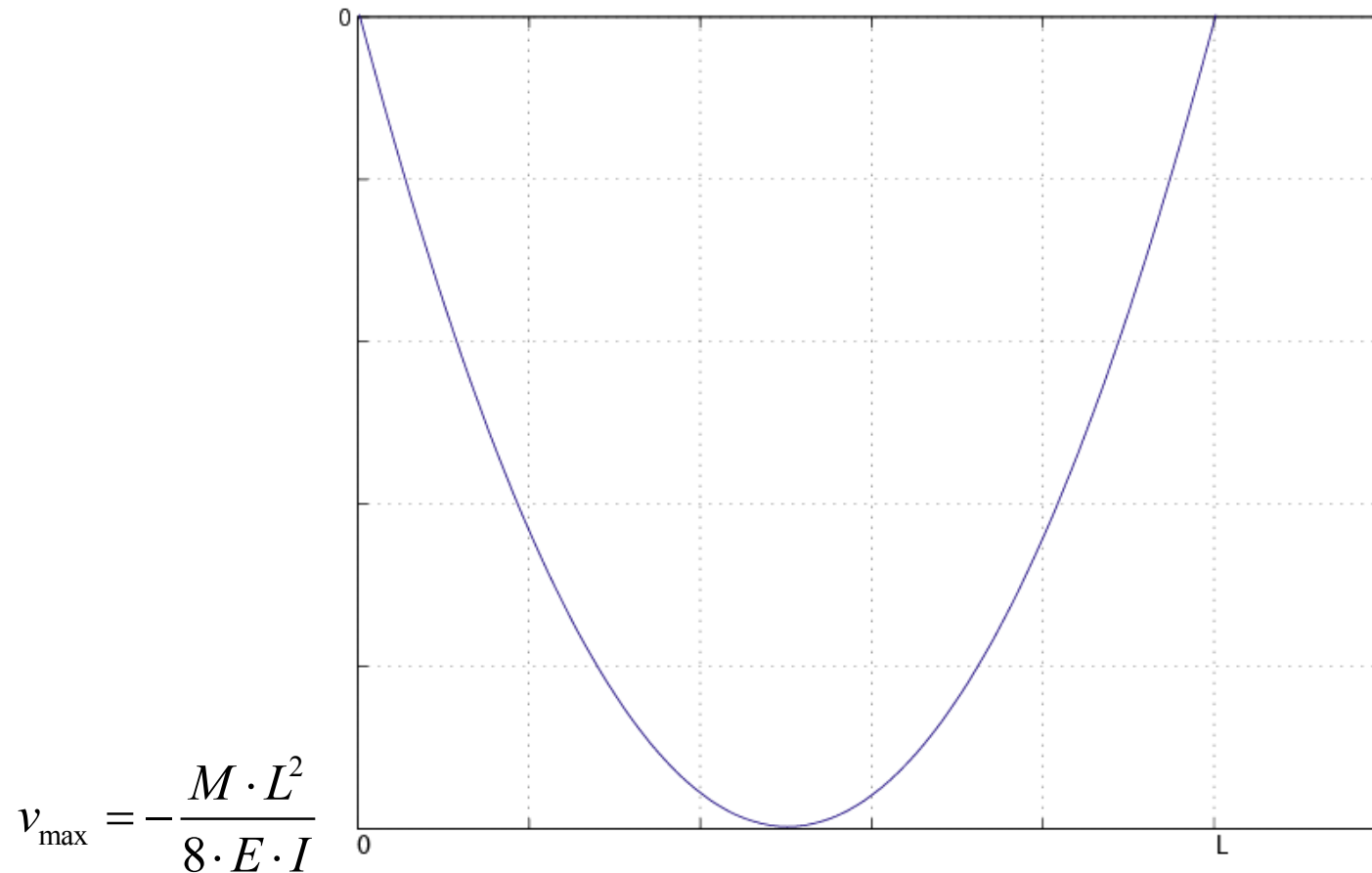
$$E \cdot I \cdot 0 = M \cdot \frac{0^2}{2} + A \cdot 0 + B \Rightarrow B = 0$$

$$E \cdot I \cdot 0 = M \cdot \frac{L^2}{2} + A \cdot L \Rightarrow A = -M \cdot \frac{L}{2}$$

$$\Rightarrow v = \frac{M}{2 \cdot E \cdot I} (x^2 - x \cdot L)$$

$$\Rightarrow \varphi = \frac{dv}{dx} = \frac{M}{2 \cdot E \cdot I} (2 \cdot x - L)$$

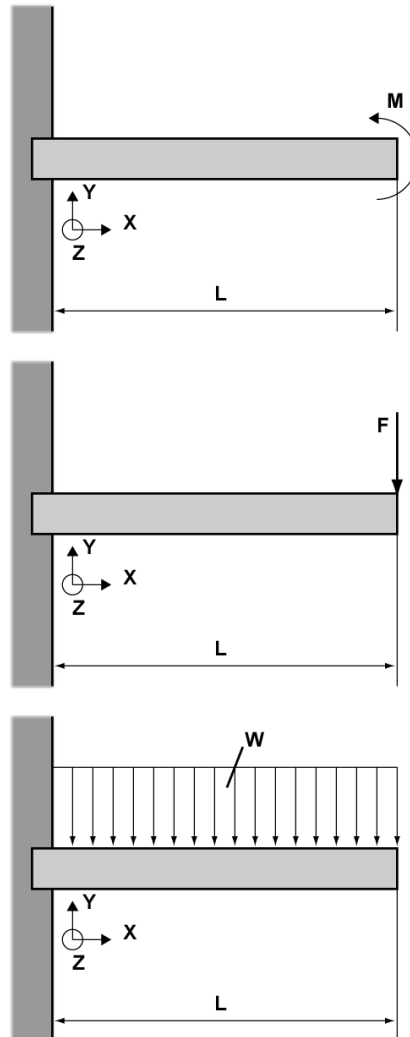
Deflection graphically



Be careful: deflection amplitude highly exaggerated!

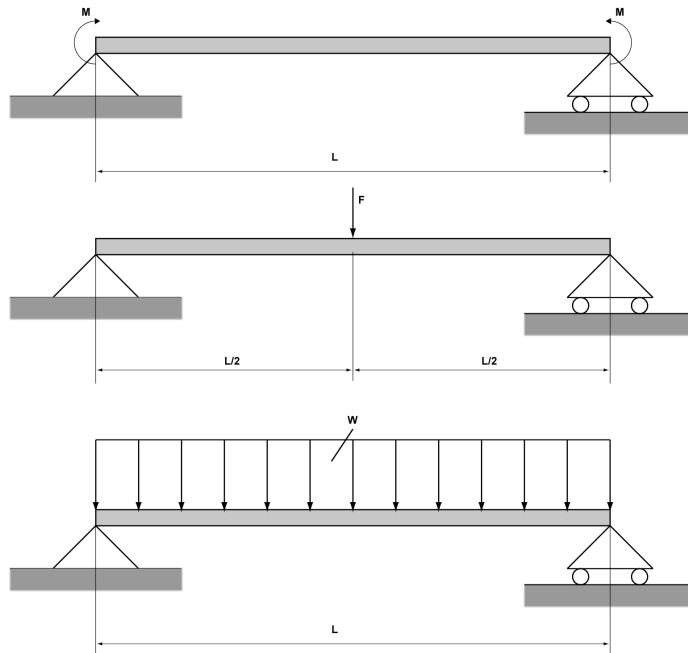
Bending deflection cantilever

Assumption: beams are weightless



Load case	End deflection in m	End slope in radians
M	$v = \frac{M \cdot L^2}{2 \cdot E \cdot I}$	$\varphi = \frac{M \cdot L}{E \cdot I}$
F	$v = \frac{-F \cdot L^3}{3 \cdot E \cdot I}$	$\varphi = \frac{-F \cdot L^2}{2 \cdot E \cdot I}$
w	$v = -\frac{w \cdot L^4}{8 \cdot E \cdot I}$	$\varphi = -\frac{w \cdot L^3}{6 \cdot E \cdot I}$

Bending deflection supported beams



$$v_{central} = \frac{-5 \cdot W \cdot L^4}{384 \cdot E \cdot I}$$
$$\varphi_{end} = \pm \frac{W \cdot L^3}{24 \cdot E \cdot I}$$

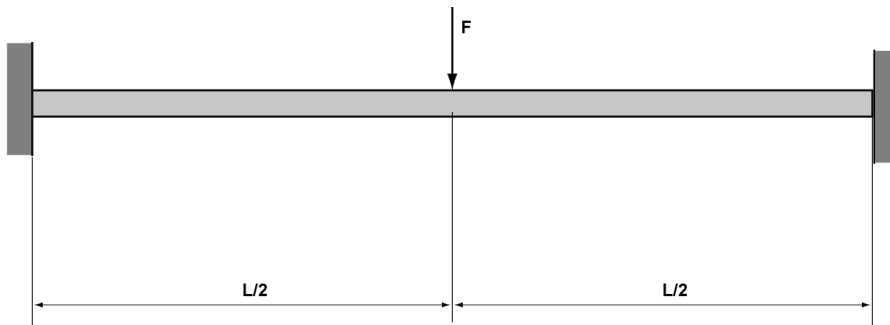
Assumption: beams are weightless

$$v_{central} = \frac{-M \cdot L^2}{8 \cdot E \cdot I}$$
$$\varphi_{end} = \pm \frac{M \cdot L}{2 \cdot E \cdot I}$$

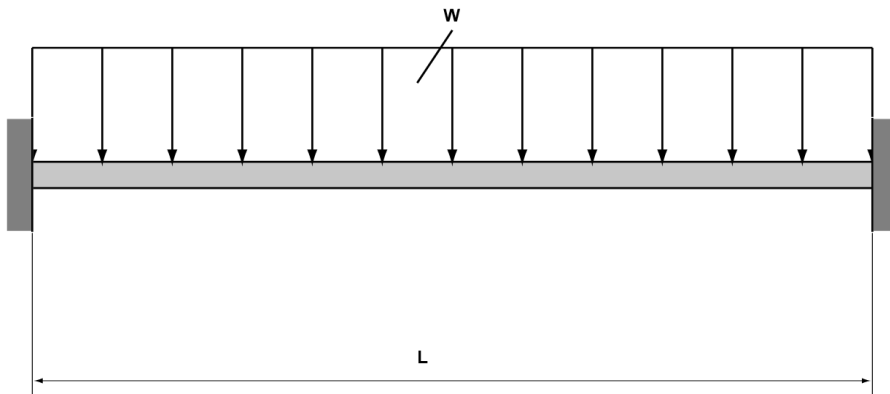
$$v_{central} = \frac{-F \cdot L^3}{48 \cdot E \cdot I}$$
$$\varphi_{end} = \pm \frac{F \cdot L^2}{16 \cdot E \cdot I}$$

Bending deflection built-in beams

Assumption: beams are weightless



$$v_{central} = -\frac{F \cdot L^3}{192 \cdot E \cdot I}$$
$$M_{end} = \frac{F \cdot L}{8}$$



$$v_{central} = -\frac{W \cdot L^4}{384 \cdot E \cdot I}$$
$$M_{end} = \frac{W \cdot L^2}{12}$$

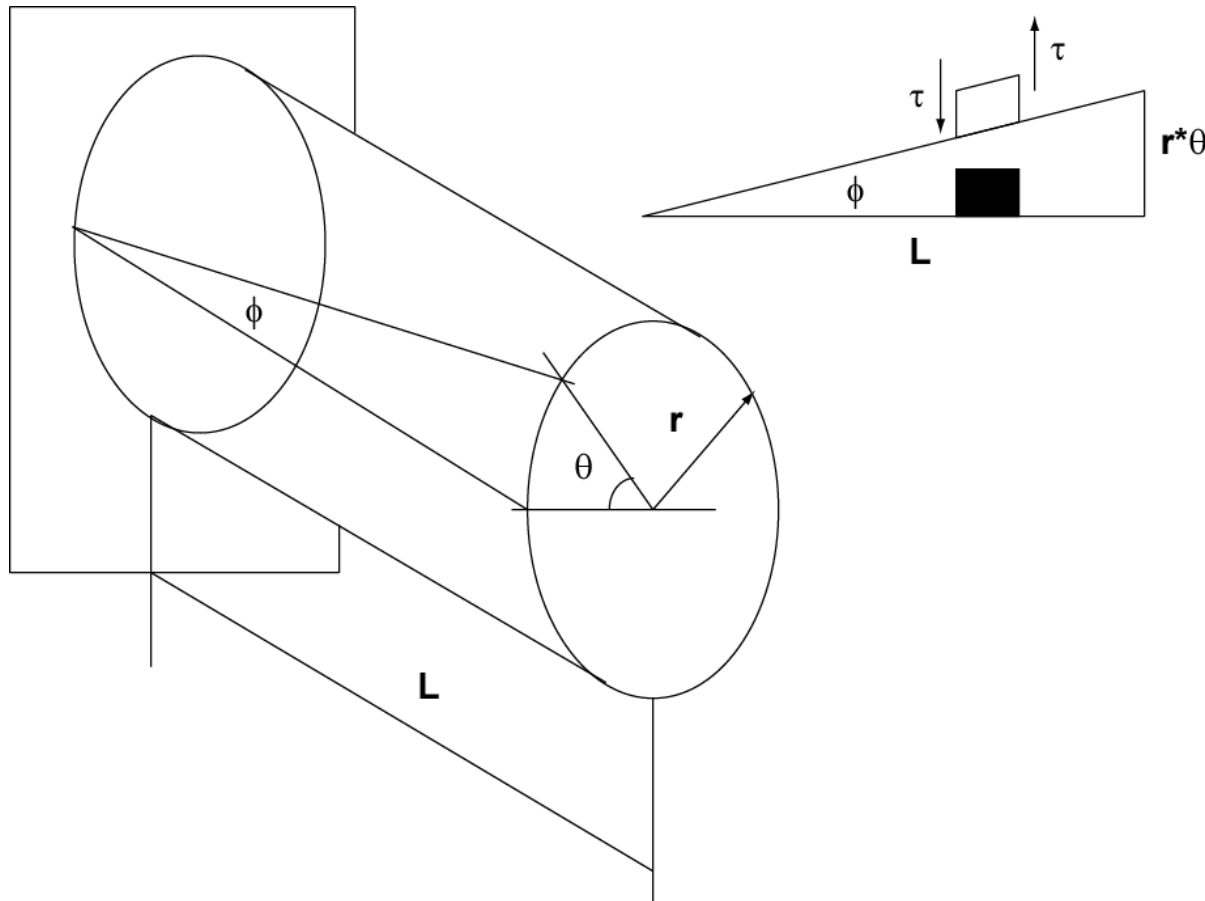
How to reduce bending deflection?

- **E modulus higher**
 - For instance from Al go to Steel, disadvantage: St has a higher specific mass
- **I, moment of inertia, higher**
 - More material at the outer fibers
- **L smaller, if possible!**
- **Reduce loads if possible**

Torsion deformation

- Due to torsion
- How to reduce?
- Examples

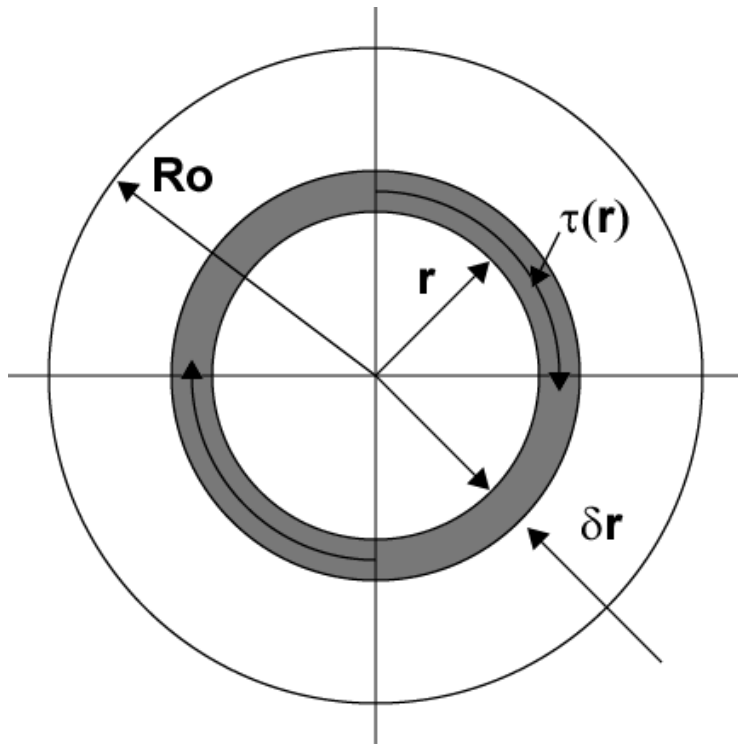
Torsion deformation



$$= \tan \phi = \frac{r \cdot \theta}{L}$$

$$= G \cdot \gamma(r) = \frac{G \cdot r \cdot \theta}{L}$$

Torsion (cont.)



$$\delta M = \tau \cdot (2\pi \cdot r \cdot \delta r) \cdot r = 2\pi \cdot \tau \cdot r^2 \cdot \delta r$$

$$M = \int_0^{R_0} 2\pi \cdot \tau \cdot r^2 \cdot dr$$

$$\tau = \frac{G \cdot r \cdot \theta}{L}$$

$$M = \frac{G \cdot \theta}{L} \int_0^{R_0} 2\pi \cdot \tau \cdot r^3 \cdot dr = \frac{G \cdot \theta}{L} \cdot \frac{\pi \cdot R_0^4}{2}$$

$$J = \frac{\pi \cdot R_0^4}{2}$$

$$\Rightarrow \frac{M}{J} = \frac{G \cdot \theta}{L} = \frac{\tau}{r}$$

$$\Rightarrow \theta = \frac{M \cdot L}{J \cdot G}$$



Finishing up

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Where innovation starts

Designing for static load bearing

- Failure mechanisms to consider

Designing for stiffness

**Highest stiffness
achievable: E modulus!!**

- Thus small deformations and low vibrations wanted!
- What to consider
 - Parallel stiffnesses:
 - Serial stiffnesses (the chain is as strong as its weakest link):

$$K_{v_{par}} = \sum_{i=1}^n K_i$$

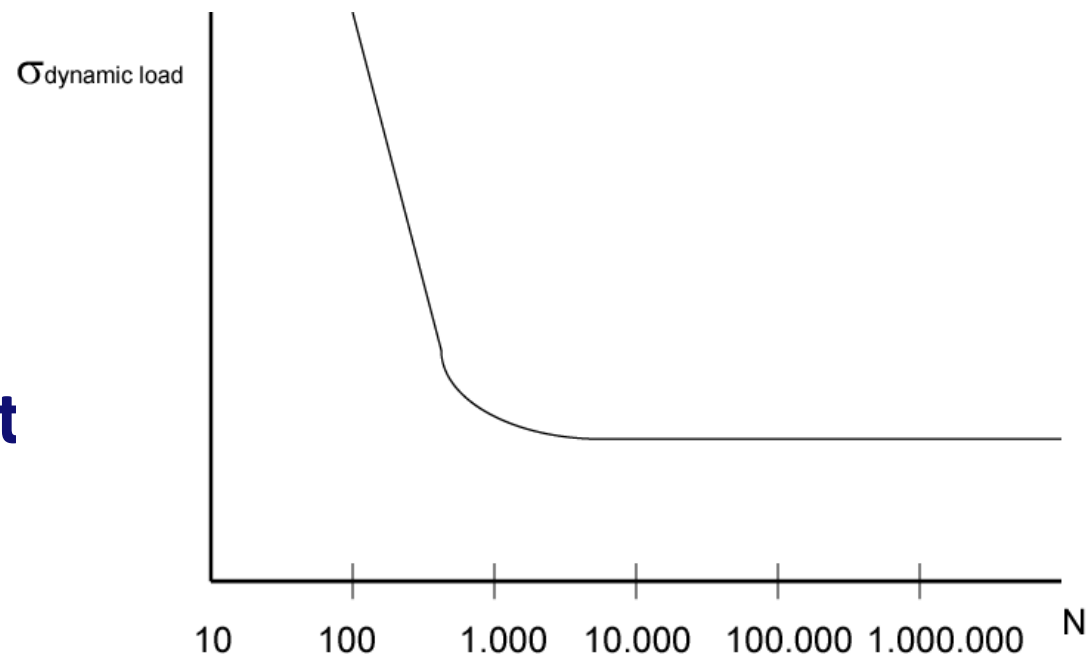
$$\frac{1}{K_{v_{serial}}} = \sum_{i=1}^n \frac{1}{K_i}$$

Possible problems in mechanical constructions:

- **Dynamic loading**
 - Be careful with fatigue limits
 - Wohler curves (Max. allowable stress is a function of the number of cycles)
- **Thermomechanical loading**
 - Thermal material limitations, especially for polymers!
 - Important for precision applications due to thermal deformations

Designing for dynamic load bearing

- Failure criteria
- Fatigue strength
 - Wohler curves
- Aluminum has no fat strength it always breaks!

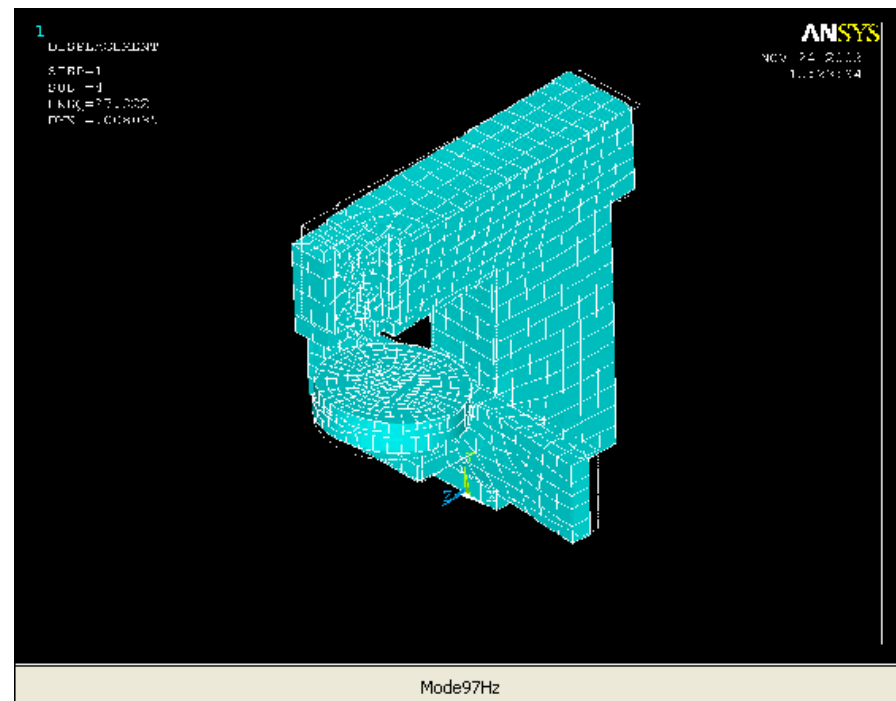
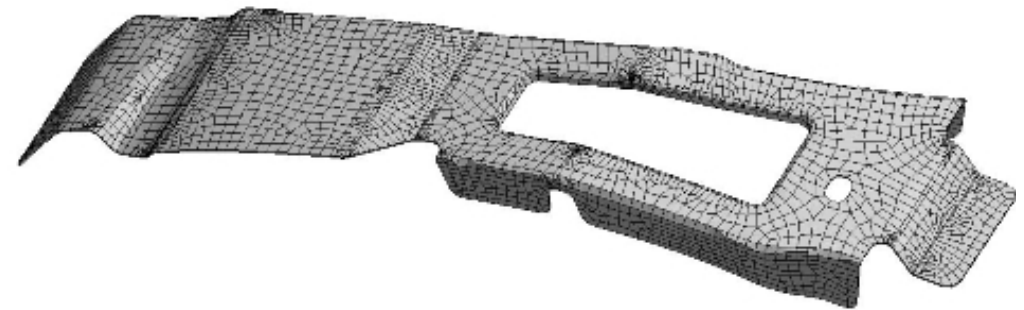
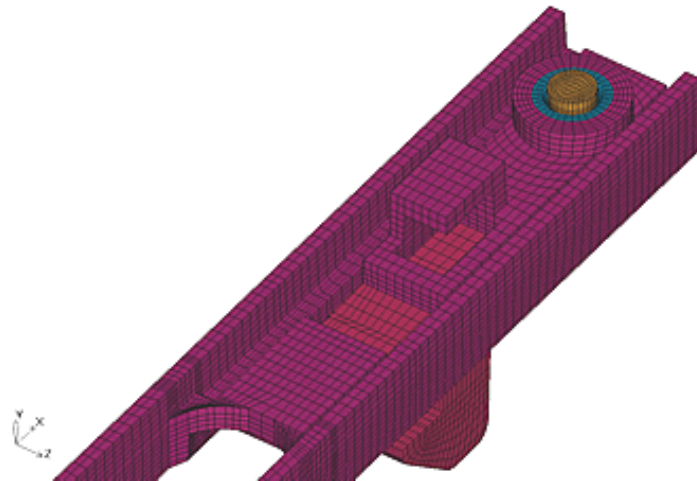


Fatigue: infamous example



4-28-1988 After 89,090 flight cycles on a 737-200, metal fatigue lets the top go in flight.

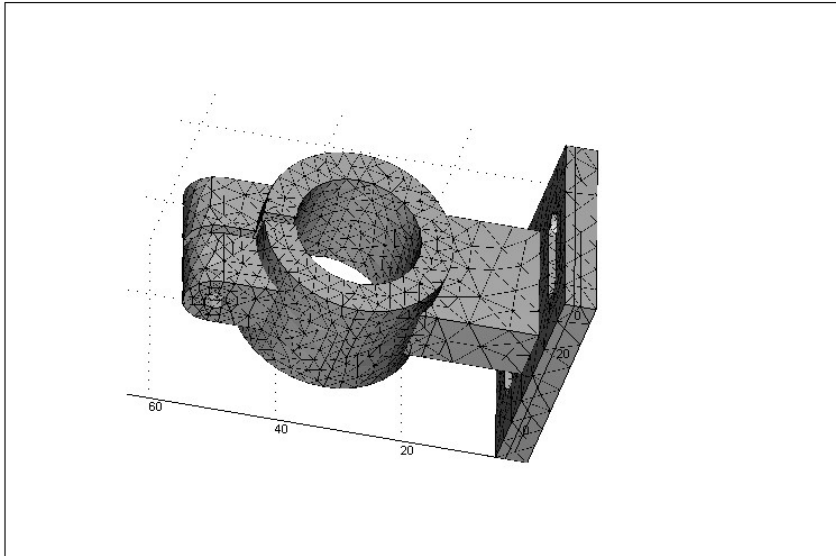
FEA, Finite Element Analysis



FEA, basic principles

- **Divide object in connected elements**
 - Each element is relatively simple (example triangles)
- **Define loads**
- **Define boundary conditions**
- **Calculate stresses and/or deformations for each element**
- **Calculate stresses and deformations of the whole model by walking through the connected elements**

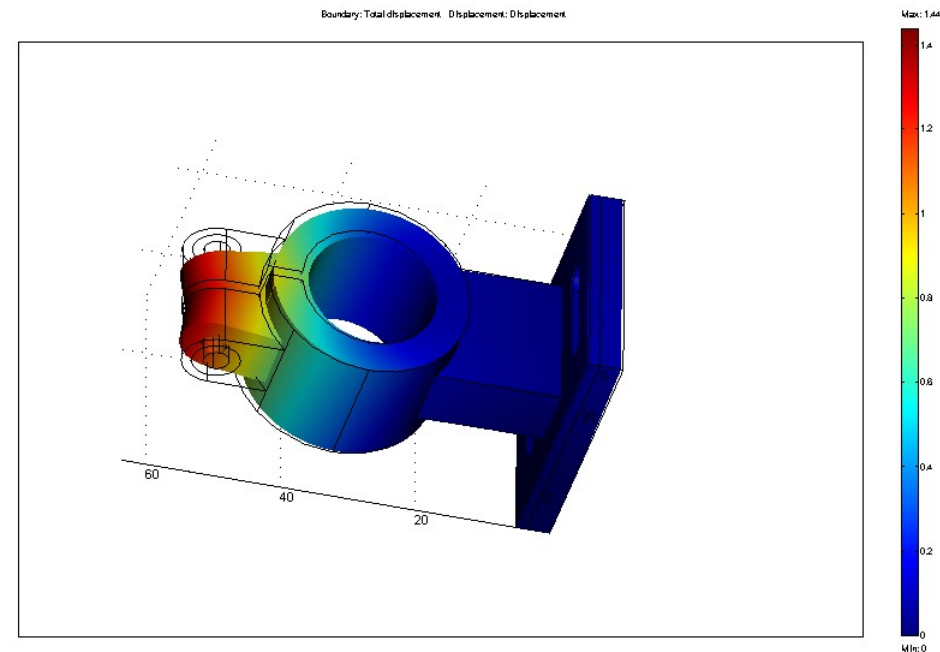
Example from FEMLAB



Element distribution.
Notice the element
distribution differences

Shown are:

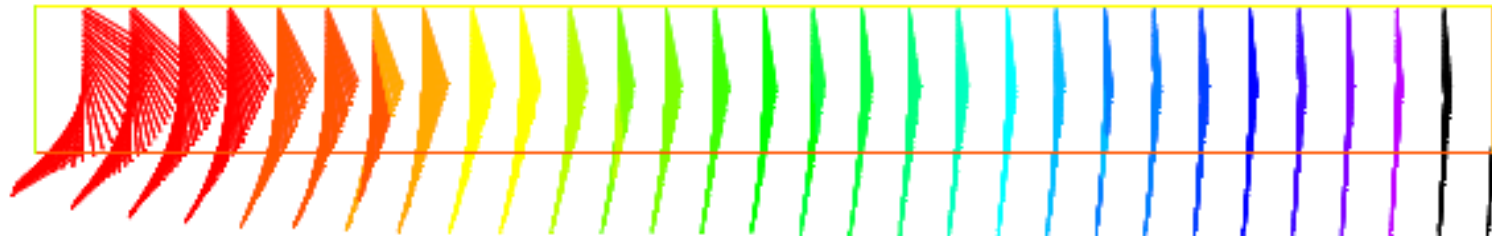
- deformations
- stresses (colors)



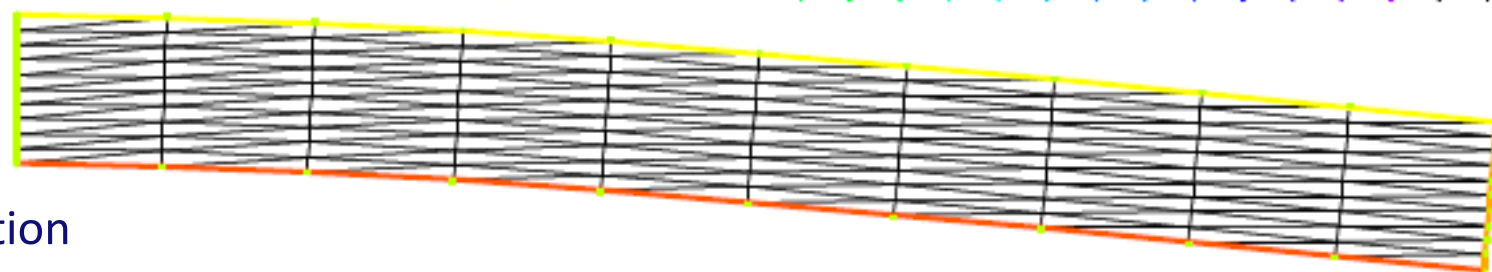
Try Finite Element Method

- www.freefem.org/ff++/index.htm
- Try for instance: lame.edp

Displacement vectors



Deformation



Summary mechanics

- 1. Determine geometry, dimensions and materials**
 1. Length, cross-sections, Young's modulus, Yield strength, etc.
- 2. Determine loads**
 1. Forces, moments and locations
- 3. Determine boundary conditions**
 1. Free, fixed, clamped and locations
- 4. Calculate force graphs for each beam/part**
 1. Tension
 2. Compression, check critical Buckling force!
 3. Shear
- 5. Calculate moment graphs for each beam/part**
 1. Bending
 2. Torsion

Summary mechanics (cont.)

6. Calculate moment's of inertia
7. Calculate stresses (and strains)
 1. Tension/compression, shear, bending, torsion
 2. Beware of stress concentrations
8. Compare the stresses with the allowable stresses of the material (Often Yield strength with safety factor, max. shear stress = Yield strength/2)
9. Calculate the deformations
 1. Tension/compression, shear, bending, torsion
10. Compare the deformations with the allowable deformations

List of symbols

F = Force in N

M = Moment in Nm

E = Young's modulus in N/m^2

G = Shear modulus in N/m^2

ε = Strain in -

γ = Shear strain in -

σ = Stress in N/m^2

τ = Shear stress in N/m^2

I = Moment of inertia in m^4

v = Deflection in m