Divisions of the Plane by Computer: Another Way of Looking at Mondrian's Nonfigurative Compositions

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ABSTRACT

The article discusses a novel way of looking at Mondrian's nonfigurative paintings. Different periods of Mondrian's life correspond to distinct types of nonfigurative compositions, but can the distinction be formalized? How many bits or numbers are needed to characterize a typical composition? Can the rules of a composition type be expressed in the language of the computer? If distinct composition types require different computer programs, can these be based on a common framework, a mechanism, perhaps? The findings presented here are only tentative, but it is interesting to note that some characteristics can be modeled reasonably well, whereas others still resist formalization in the presented framework. The author's approach borrows principles from genetic programming. Employing a built-in random number generator, it can be used to explore a large space of "compositions."

I shall begin with a survey of relevant analyses and formalizations of Piet Mondrian's nonfigurative works [1]. In the sections thereafter I present a step-by-step description of a novel approach.

Analyses and Formalizations of Mondrian

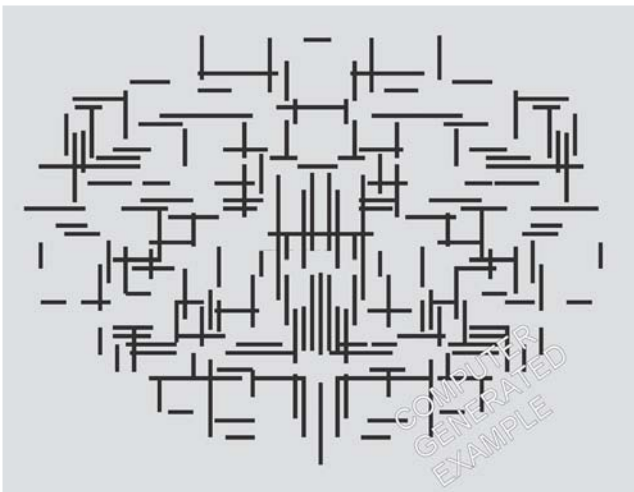
Schufreider [2] discusses the role of the grid in Mondrian's work. The grid, in particular the Cartesian grid, is "the very emblem of space in Modernism." According to Schufreider, Mondrian employed the grid as an underlying and stabilizing structure (e.g. in the checkerboard works). This type of grid is an unresponsive structure. Later, Mondrian developed the grid to serve another function: a structure of openness. An "interplay of difference" emerges: Each element contributes to the overall order without any one taking control. The grid loses its dominance and there is instead a "complex of relationships" at work among the various elements. Philosophically, Schufreider concludes that Mondrian offers an opening to postmodernism.

In 1964 A. Michael Noll [3–5] made a computer program to mimic Mondrian's 1917 *Composition with Lines*. Noll varied the degree of randomness, which ranged from a grid-based placement of varying-length lines to a completely random placement. He compared the real Mondrian to his generated work by showing both to 100 people from a selected audience. The majority preferred the computer version and believed Mondrian had created it. This and other programs by Noll marked the beginning of computer art as a branch of the autonomous arts—not as a tool for studying existing works.

Several Java applet Mondrian composition generators can be found on the Web. The example at <javaboutique.internet.com/Mondrian/> seems to best fit the 1937 period, generating both crossing and noncrossing lines. Bielak's Mondrian Creator at <www.netlabs.net/hp/richieb/java/Mondrian.htm> varies the line thickness but produces only noncrossing lines. Linhart's MONDRIMAT at <www.stephen.com/mondrimat/mondrian> works interactively. A plane is split at each step, in a recursive manner. A list of "create your own Mondrian" web sites is available at <www.geocities.com/piran_montford/mondrian/>. An overview of tests for distinguishing real Mondrians from randomly generated compositions can be found at <www.snap-dragon.com/articles.htm> and in *Nature* [6].

Carel Blotkamp [7] offers an illuminating view on the composition principles deployed by Mondrian, which he calls "the Art of Destruction." Mondrian wanted his compositions to have an aesthetic quality of their own, without reference to figuration. He sought to hide the object-character (individuality) of the compositional elements and avoid those interactions between elements that reintroduce three-dimensional effects. For example, the doubling of lines serves to hide (destroy) the individual forms of the lines. For another example, gray or black lines separating the color planes serve to prevent the impression that the color planes float in front of a white background, which is a "problem" with, for example, *Composition with Color Planes 2*.

Fig. 1. Short black lines (made by computer). (© Loe Feijs)



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MONDRIAN'S PAINTINGS

The work of Mondrian (1872–1944) ranges from beautiful figurative paintings to the nonfigurative abstract compositions that made him famous. He followed a process of increasing abstraction, working along parallel threads. In the period 1910–1920, he restricted his topics to windmills, ocean coasts, trees, church towers and flowers. Except for the flowers, these topics were subjected to experiments of further abstraction. He adopted a restricted set of colors, sometimes soft pastels, eventually only red, yellow and blue, next to black, white and sometimes gray. He reduced the number of compositional elements until only horizontal and vertical elements remained. He removed most of the texture and shading from the color planes. Eventually this left him with a set of compositional elements that were pure and simple yet spanned a rich search space for exploring the essentials of composition. The most important of these elements are planes, lines and colors.

Examples of Composition Types

Planes. The planes as individual elements, in (soft pastel) colors, appear already as abstract clouds in Mondrian's figurative paintings such as *Windmill in Sunlight* (1908), *Dune II* (1909) and *Church Tower in Zealand* (1911). In certain later nonfigurative paintings they are the sole elements of composition, notably in the *Composition with Color Planes 2* (1917) and in four similar paintings. Compare the computer-generated bitmap shown in Color Plate B No. 2 with Mondrian's *Composition with Color Planes 2* (1917) (later I explain precisely how I made the computer-generated images).

Color Plate B No. 2 does not have the aesthetic qualities of the real *Color Planes 2* (1917). So why does this not work? What is wrong? Looking again at *Composition with Color Planes 2*, a difference in orientation becomes apparent: This type of composition works best with a canvas that is oriented horizontally. Strangely, some planes seem to be slightly tilted. This is an optical illusion, however, since in the computer bitmap they are perfectly upright. This optical illusion appears in the real works, too. Also, the interplane distance should be slightly increased. In this way the bitmap is an invitation to have another look at the real work, which in its turn reveals compositional aspects easily overlooked otherwise.

Lines. Mondrian derived the first nonfigurative works with line segments as the sole element of composition from ocean coasts, trees and church towers. *Pier and Ocean 1–4* (1914) are intermediate between figurative and nonfigurative works. The pier is still there. But *Composition 10 in Black and White, Pier and Ocean* (1915) is close to nonfigurative. Compare it with the bitmap shown in Fig. 1.

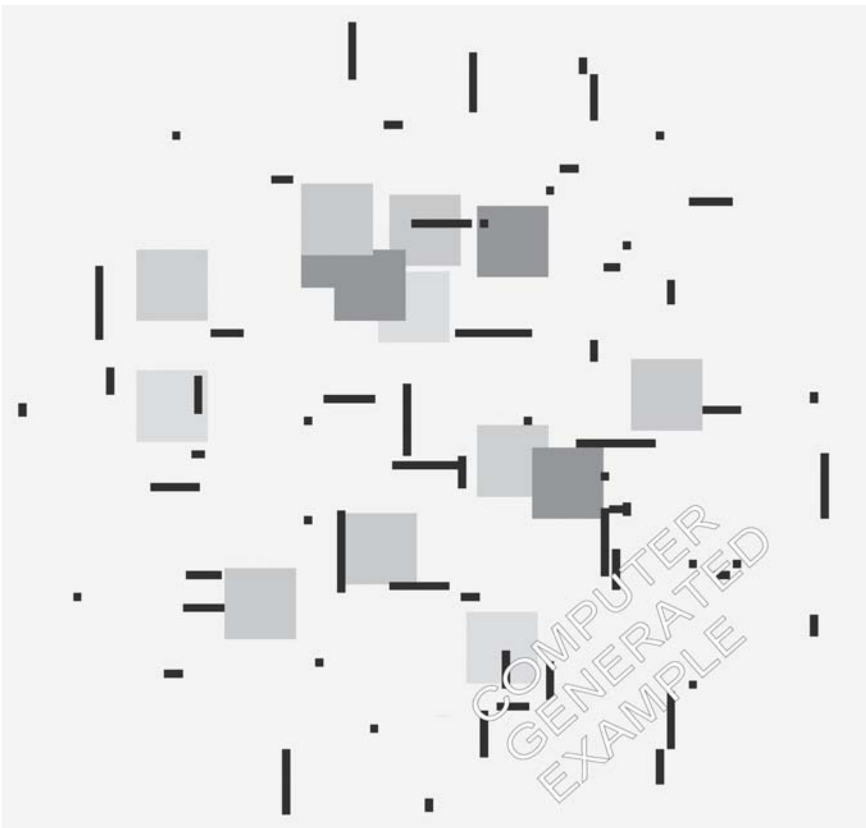
Several characteristics of the intended composition type are modeled reasonably well: Many elements appear symmetrical, although there is no full symmetry. The longer vertical lines below the center are typical too. Typical also is that many, though not all, vertical lines stop at a horizontal line, and many horizontal lines stop at a vertical line. Other characteristics need tuning: The number of lines needs to be increased to model Mondrian more successfully. *Composition 10 in Black and White, Pier and Ocean* has a dark gray background except for the ellipse containing the lines. There are subtypes corresponding to the origins: ocean coasts, trees and church towers. *Composition in Line* (1917) is the most abstract of these works. Its circular area is intermediate between the horizontally oriented ellipses of the ocean coast type and the vertical orientation of the church towers. At the detail level, it lacks the symmetry of Fig. 1. Moreover, it is no longer possible to tell whether *Composition in Line* (1917) stems from an ocean coast or a church tower. Mondrian integrated the lessons of both types and removed the last references to the subject of the painting.

Combined usage of planes and lines. The first type of nonfigurative composition in which Mondrian used planes and lines combines the elements and the methods of composition from the above-mentioned plane composition and line composition types. Compare a work of the type seen in *Composition in Color B* (1917) with the computer-generated bitmap shown in Fig. 2 (again, later I explain precisely how I made it).

How to judge Fig. 2? It lacks the aesthetic qualities of the real *Composition in Color B*. But are the planes and the lines composed in the right way? Is it not strange that the planes overlap? Perhaps the spacing approach of Color Plate B No. 2 is better than allowing planes that intersect. Inspection of *Composition in Color B* (1917) shows that some planes indeed overlap. The lines-planes relationship seems mostly based on the approach described for line compositions. So these characteristics are modeled reasonably well.

Mondrian combined planes and lines in another way in his checkerboard works. The lines are equidistant and mostly of equal weight. The main degree of freedom left is the choice of colors for

Fig. 2. Colored planes and short black lines (made by computer). (© Loe Feijs)



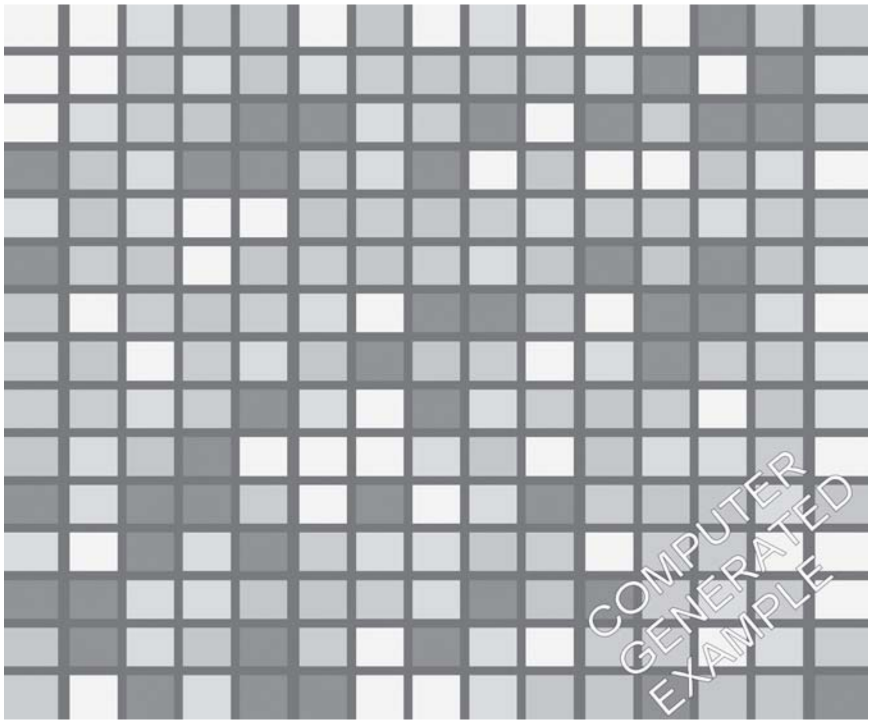


Fig. 3. Planes and lines forming a checkerboard (made by computer). (© Loe Feijs)

the enclosed planes. An example is *Grid Composition 8* (1919) with light colors. Letting a computer generate checkerboards is easy; see Fig. 3.

Mondrian created two more types with combined usage of planes and lines by relaxing the regularity constraints (as in *Diamond-Shaped Composition 5* [1919], which has 16 × 16 rectangular planes) and finally dropping the regularity constraints altogether (for example in *Composition A* [1920] in the colors black, red, gray, yellow and blue). Mondrian kept one compositional aspect from the checkerboards and largely adhered to it for almost 20 years: The dark lines separate the colored planes. Compare *Composition A* (1920) with Fig. 4.

Is the bitmap seen in Fig. 4 "right"? Several characteristics of the type exemplified by *Composition A* are modeled reasonably well. There is a random distribution of the main colors black, gray, red, yellow and blue (white is used for the background color). The black lines do not form a grid because sometimes a black line stops upon meeting another black line, whereas on other occasions the black lines cross. Two black lines do not run to completion but back off at a certain distance from the border of the figure.

Computer Model

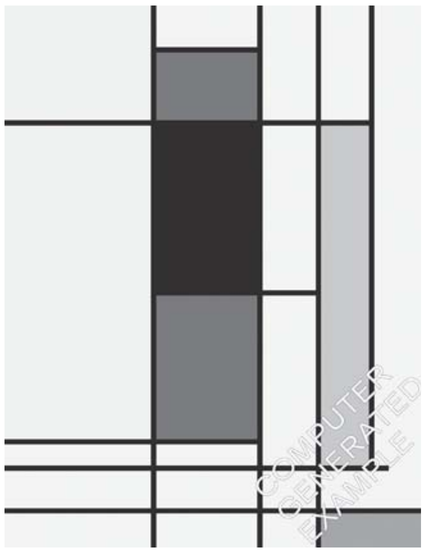
Color Plate B No. 2 and Figures 1–4 are made by a computer program, which I explain here. The program works in essentially the same way for all types. Each specific division of the plane, e.g. see Fig. 4, is described by parameters. A type defines a range for each parameter. For a given type, each parameter is chosen by a random generator, which respects the type's range restrictions. The most important parameters are *A*, the number of lines, and *B*, the number of planes. For the type shown in Color Plate B No. 2 they are given by $A = 0$ and $15 \leq B \leq 35$. Further parameters concern individual lines and planes. Lines and planes are essentially treated the same. So-called cells represent them. So in Color Plate B No. 2 there are $A + B = 0 + 24 = 24$ cells. For example, for the type of Color Plate B No. 2, the color of the (i+1)-th cell $C[i]$ is restricted by $C[i]$.*color* ∈ {*rose, sky, gold*} for all *i* such that $0 \leq i < A + B$. For the type of Fig. 4 the color of the planes is restricted by $C[i]$.*color* ∈ {*yellow, red, blue, black, gray*} for all *i* such that $A \leq i < A + B$ (assuming white to be the background color).

I cannot generate the coordinates of the cell boundaries by letting a random generator deliver values in a specified range. The coordinates of adjacent cells depend on each other. For example, in many types, the rightmost boundary of a vertical line has to coincide with the leftmost boundary of one or more planes. *Composition with Color Planes 2* (1917) and Color Plate B No. 2 demonstrate also that the colored planes can depend on each other.

To overcome this problem, I chose a dynamic approach, letting cells expand. The random generator determines only an initial pair of *x,y* coordinates for each cell. In Figs 2 and 3 these positions are constrained to lie in an ellipse; in Fig. 4 they are distributed over the entire plane. A cell grows from its initial position in all four directions until it reaches its length limit or it approaches either another cell or the outermost boundary. For example, in the types seen in Figs 1 and 2, most cells stop growing because of the (randomly chosen) length limit, whereas in the type shown in Fig. 4, where the length limits are set to infinity, the *A* cells stop only at the outermost boundary. Upon approaching another cell, the behavior depends again on the cell's growth parameters: If it is a cell whose *crossing* parameter equals *true,* then it continues to grow; otherwise it stops at a random distance from the neighbor. In Color Plate B No. 2, all the plane cells have *crossing* = *false.* In Fig. 3, four horizontal line cells and one vertical have *crossing* = *false,* whereas two horizontal and three vertical lines have *crossing* = *true.*

The difference between cells that become planes and cells that become lines is governed by another pair of cell parameters, called *xInc* and *yInc.* A cell with *xInc* = *true* has its size in the *x* direction specified incrementally during development: It grows horizontally, both to the left and to the right. A plane has *xInc* = *true* and *yInc* = *true.* A horizontal line has *xInc* = *true* and *yInc* = *false.* A vertical line has *xInc* = *false* and *yInc* = *true.* The computer program is built around a generic growth engine, interpreting the growth parameters for each cell in an iterative way. To complete the

Fig. 4. Colored planes separated by black lines (made by computer). (© Loe Feijs)



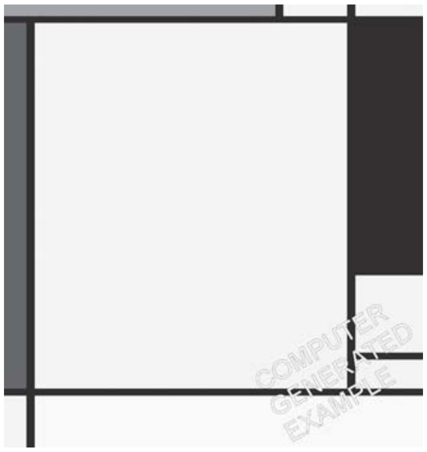


Fig. 5. Peripheral planes and lines (made by computer). (© Loe Feijs)

picture, as it were, each cell has a *phase*, which is a number between 0 and 15. The higher the phase, the later the engine initiates the growth of the cell. This is shown in Figs 3 and 4: the lines have *phase* = 0, so they complete first, and the planes develop afterward. Planes have a random *phase* between 1 and 15.

I distinguish between the genotype and phenotype of a cell. The parameters *color, x, y, phase, xInc, yInc, crossing, maxLength* (length limit), *d* (size of the initial square), *nBackOff* (behavior near a neighbor) and *bBackOff* (behavior near boundary) determine the genotype. The cells' environment during growth determines the boundaries *xMin, yMin, xMax, yMax*. Together with the color, these boundaries determine the phenotype. A composition type is described by the ranges for the numbers *A* and *B*, the parameter ranges for the *A* cells, which will turn into lines, and the parameter ranges for the *B* cells, which will turn into planes. From this a collection of $A + B$ lines and planes will develop (the color and boundaries of these are easily assembled to form a bitmap).

More Composition Types

Having explained the computer model, I resume here the discussion of Mondrian's compositions (using terminology based on my program). From 1917 onwards the elements of the compositions are stable: rectangular cells that may or may not cross each other and may or may not keep a distance ("back off") from each other or the borders. Cells appear as lines, horizontal or vertical, or as planes. From 1920 onwards, the colors are restricted to the three primary colors, appearing next to white, gray and black. Most, if not all, reference to figuration disappears. The question that remains is how to combine these elements best to get compositions of a general kind of beauty.

In 1920 Mondrian created compositions with (thin) gray lines; in 1921, a number of compositions with (massive) black lines. In 1922, the compositions here called *peripheral* appear, for example *Tableau 2, with Yellow, Black, Blue, Red and Gray* (1922). I try to characterize this composition type by defining the model parameters. The typical composition has heavy black lines, with little variation in their thickness. Four lines appear close to the outermost borders, one parallel to each border. Still closer to the borders, more lines appear, perpendicular to the border; they are grown in a later phase than the parallel lines because they run precisely up to the parallel line. A color plane fills some of the peripheral areas thus created. The central area is empty. The lines largely do not cross and do not "back off" near their neighbors, but many lines back off near the outermost border. The planes show no back-off behavior at all. Typical values are $A = 8$ and $B = 4$. The computer-generated image in Fig. 5 is obtained in this way.

The program is a tool for revealing whether the formulated characteristics are complete. It produces many bitmaps not resembling the real 1922 peripheral compositions at all. In this way I detected a missing constraint: In most 1922 peripheral compositions each color from {*yellow, blue, red, black*} appears precisely once (this applies not only to 1922 peripherals, but also to most others from 1921–1924). I also observe that some 1922 peripheral compositions are slightly more complex: the *Composition with Blue, Yellow, Black and Red* (1922) has an additional (noncrossing) line parallel to the upper border. I do not claim that the computer comes close to real Mondrian compositions; on the contrary, every desired property not formalized well is laid bare immediately. But that is precisely why the computer as a tool stimulates careful observation.

In 1925–1927 Mondrian painted many compositions that are three-sided open variations on the peripheral type, for example *Composition No. III, with Red, Yellow and Blue* (1927). In 1928 and 1929 this development is pushed further by a two-sided open variation, for example *Composition No. 1, with Red and Black* (1929). As the artist decreased the number of borders involved, he increased the size of the populated peripheral area.

The peripheral area increases until the two border-parallel lines form a cross positioned almost centrally. An example is the *Composition with Yellow and Blue* (1932). The principle of two lines forming a cross and putting the initial positions in one of the quadrants can be formalized; see Fig. 6.

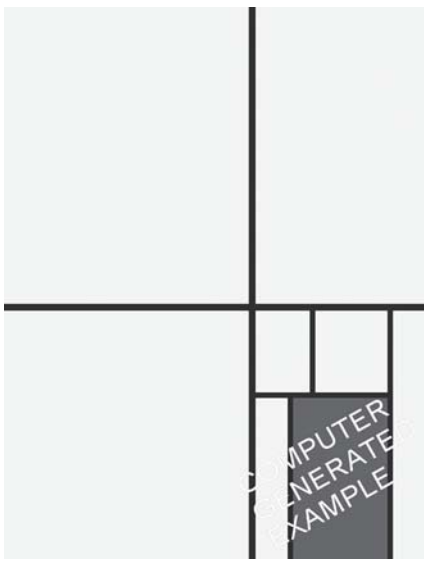
In 1932 one of the lines of the cross is doubled. In 1934–1936 more doubled lines appear, and sometimes the spacing of the doubled line becomes larger. In 1937–1939 this development is pushed further, until the concept of lines forming a centrally positioned cross does not apply anymore. There are horizontal and vertical lines ($7 \leq A \leq 13$), mostly crossing, with no back-off behavior and few planes ($B \leq 2$) (see Fig. 7). Several subtypes can be distinguished. Good examples are *Composition—Blanc, Rouge et Jaune: A* (1936) (only peripheral color planes) and *Rythme de Lignes Droites* (1935–1942) (also nonperipheral planes, sometimes forming one-color clusters, as a kind of superplane).

A new compositional element appears in the period 1940–1943: short, mostly noncrossing colored lines, typically in peripheral areas. An illustrative example is the *Place de La Concorde* (1938–1943). Although I can model this type, most results are ugly. In 1941–1942 a completely new type of composition appears, containing a large number of crossing colored lines; the difference between lines and planes has disappeared. An example is *New York City* (1942). This type fits my computer model easily.

Evaluation

Several aspects can be modeled reasonably well in the given framework. The

Fig. 6. Two black lines forming a cross (made by computer). (© Loe Feijs)



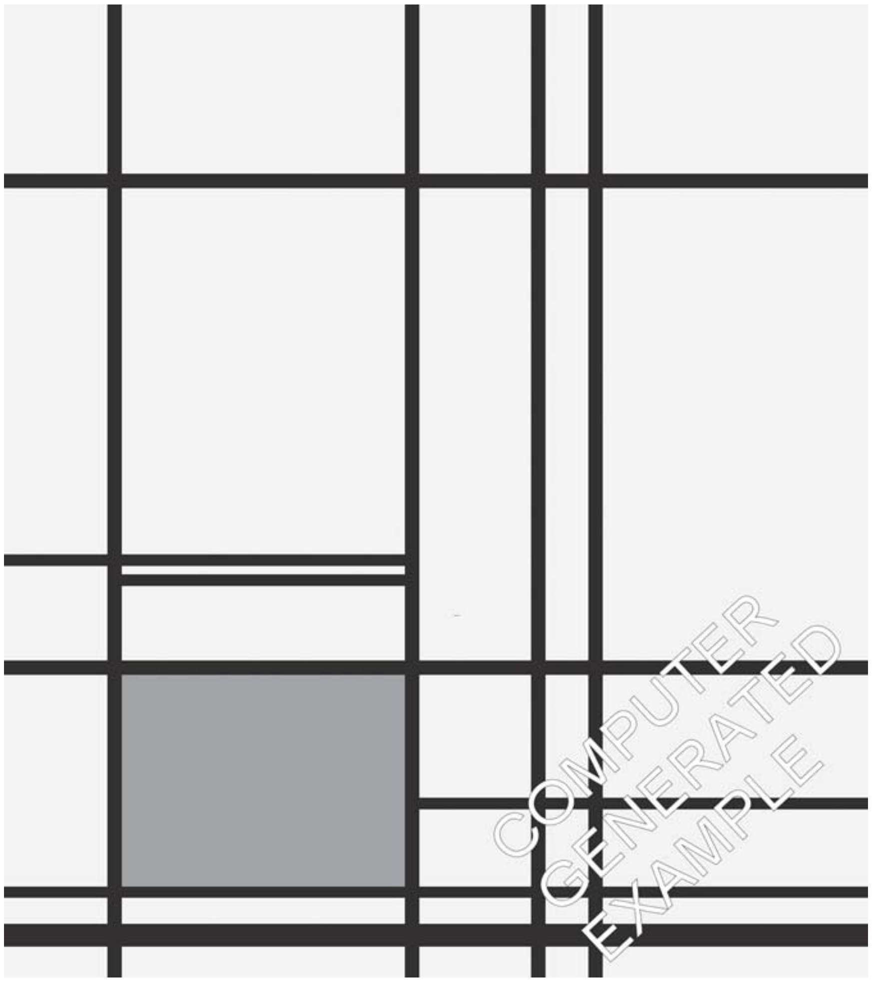


Fig. 7. Mostly crossing lines, few color planes (made by computer). (© Loe Feijs)

time-period of Mondrian's development thus covered is more than 25 years. In my study I did not attempt to model the semi-regular grids (such as *Grid Composition 7, Diamond-Shaped* [1919]). They do not fit into the presented computer model; they seem based on a checkerboard in which small planes are merged into larger planes. I did not try to model the diagonal lines (as in the *Grid Composition 1* [1918]) or the diamond-shaped canvases (such as the *Composition with Two Lines* [1931]). I expect that the diamond-shaped canvases as such do not really pose new problems. The "Boogie Woogie" works are not dealt with (they are quite complex, and each is unlike anything else, so there is no basis for a type definition).

There are two types that I expected to fit in the presented framework but that actually resist proper formalization. The first type of compositions is that of *Composition C* (1920) and *Composition No. VI/Composition No. II* (1920) (early compositions with black lines demarcating colored planes). I found it difficult to approximate these by computer. There is a hard-to-catch regularity: The squareness of several planes in *Composition C* (1920) resembles the square colored planes of *Composition No. 3* (1917), but, unlike in a checkerboard, I cannot choose the genotypes to let cells make squares. Secondly, I find the type exemplified in *Place de La Concorde* (1938–1943) hard to formalize because of the regularities in the short colored lines (for example, the two aligned reds in *Place de La Concorde).*

Comparison to Related Work

Regarding Schufreider's discussion of the grid and the approach of the present paper, the model underlying my computer generation provides a concrete interpretation and a more specific vocabulary for the aforementioned "complex of relationships." Indeed, the later grids act as responsive structures since, depending on the *crossing* parameter, a line may end upon approaching another element. Another remark by Schufreider concerning early tree paintings is interesting too: that there is a theological aspect to Mondrian's fascination with trees. The tree stands for a metaphysical order that moves and grows and develops. My model shows that many more paintings can be understood in terms of growth. Schufreider also discussed the Boogie Woogie works and "Wall Paintings," which, however, are beyond the scope of my work.

In comparing Noll's study with the approach of the present paper, one can see that Noll started from a grid, deviating from it by adding randomness, instead of constructing the grid through the interaction of its elements. Moreover, Noll focuses on a single composition type, not on the development of types.

Comparing Blotkamp's book with the approach of the present paper, I note that the former is analytical, whereas the latter is synthetic (generative). There is another difference: Blotkamp's explanation includes knowledge about human perception: for example, the fact that people "see" 3D effects. My computer program is of a merely syntactic nature, and its only built-in knowledge is two-dimensional. This is an advantage for developing or (later, perhaps) refining the program; it forces me to study and describe composition types in a very syntactic way. As such, my approach has no power of explanation—in contrast to Blotkamp's. The distinction is important because it demonstrates that there are at least three possible levels of describing composition types: syntactic (lines/planes), perceptive (seeing or not seeing lines/planes/3D effects) and semantic (seeing or not seeing trees/churches/seas). My approach helps to understand that the first two levels do not coincide. In terms of this three-level model, Blotkamp can be said to show that Mondrian's nonfiguration is not just the absence of semantics, but also the absence of options for perceptive interpretation.

The program cannot judge the aesthetic value of a bitmap, which depends, amongst other characteristics, on its pictorial balance. Arnheim discusses balance for combinations of squares and rectangles in *Art and Visual Perception* [8].

Schapiro [9] notes similarities in figurative works by Degas, Bonnard and Monet, notably the grid and its imaginary extensions. The canvas intercepts the grid, thus creating an explicit viewpoint. My formalization, although not in conflict with Schapiro's remarks, introduces another idea concerning the grid: that it can be obtained through a process of growth in which lines and planes are similar. Schapiro also discusses randomness and believes Noll is right in associating randomness with an idea of the creative and assuming it has a positive quality in art.

Concluding Remarks

I now return to the questions posed at the beginning of this article. Q: *Can the distinction between the types of nonfigurative compositions be formalized?* A: Yes, at least up to a certain level of approximation. Q: *How many bits or numbers are needed to characterize a typical composition?* A: For a given type and a given size, forgetting, amongst other things, the texture, from 250 to 1,000 bits. The Boogie Woogie works are of a much higher complexity. Q: *Can the rules of a composition type be expressed in the language of a computer?* A: Yes, but again only up to a certain level of approximation. My formalization provides a new way to analyze and describe the syntax of some of Mondrian's works. Q: *Is there a common framework for the computer programs involved?* A: It is possible to use a generic growth engine.

The following conclusions seem justified:

- The computer modeling of types of divisions of the plane is a useful tool for observing and classifying Mondrian's abstract works.
- The typology I discussed above is of a syntactic nature; the chosen computer modeling approach, which does not include perceptive or semantic interpretation, enforces a syntactic way of describing divisions of the plane.
- Rectangular planes and lines can be considered as two variants of the same thing and are amenable to closely related compositional principles; this seems to hold for most composition types during at least 25 years; it is possible that Mondrian used this implicitly as a guiding principle.
- The formal model confirms the intuitive observation that the early abstract works of Mondrian have a higher complexity than works in the early 1930s and that around 1940 the complexity of his work increased again.

Acknowledgments

I would like to thank Jacqueline Cove, Jan van der Lubbe, Carel Blotkamp and Kees Overbeeke for their constructive comments on the research described in this article.

Appendix A: Calculating Complexities

How many bits or numbers are needed to characterize a typical composition? The answer depends on the composition type. If I fix the type, say *checkerboard*, and estimate that each of the 16 $\times$ 16 planes is chosen from 8 colors, then assuming the grid, etc., is fixed, there are 8 *exp* (16 $\times$ 16) = 2 *exp* 768 distinct possibilities. Assuming they all have the same probability, the Shannon entropy of a random checkerboard *X* is given by $H(x)$ = 2 *log*(2 *exp* 768) = 768 bits (technical remark: the entropies thus found are upper bounds, as there can be dependencies between the two dimensions). If I adopt Kolmogorov's definition that the entropy of *x* equals the shortest program that generates *x,* I get the same answer if I consider the generic growth engine as part of the computer.

The 16 $\times$ 16 checkerboards are special, however, in the sense that the grid is fixed. For other types there is no fixed grid: lines and planes contribute to the entropy. To fix an *A*-type cell's genotype, at least 27 bits are needed (8 bits each for the *x, y* coordinates, 1 bit each for the *phase*, *xInc*, *crossing*, *nBackOff*, *bBackOff,* 3 bits each for *d*, *maxLength*). To fix a *B*-type cell's genotype, at least 22 bits are needed (8 bits each for the *x, y* coordinates, 3 bits each for *color*, *phase*). Depending on the type, these estimations need subtle corrections (e.g. a few extra bits for *B*-type *maxLength* for the 1917 *color-planes*). But for convenience of calculation, the numbers 27 and 22 are fixed for all types. The entropy of a type *T* is thus approximated by $H(T)$ = (*nr of lines*) $\times$ 27 + (*nr of planes*) $\times$ 22.

The findings after application of this theory are as follows: The complexity (= entropy) of the early compositions is high, for example 700 $\times$ 27 + 0 $\times$ 22 = 18,900 bits for an *asymmetric-pier-ocean* type and 15 $\times$ 27 + 27 $\times$ 22 = 999 bits for a *color-planes-long-black-lines type* (see *Composition A 1920*). The peripheral types have lower complexity, for example 6 $\times$ 27 + 4 $\times$ 22 = 250 bits (see *Tableau 2, with Yellow, Black, Blue, Red and Gray*). Later types have higher complexity again: 0 $\times$ 27 + 23 $\times$ 22 = 506 bits for a *colored-crossing-lines* type (see *New York City*).

Appendix B: Program Details

The program is in Turbo Pascal. I wrote the first version in 1993. It is 4,500 lines of code. I show the main case statement: a simplified timeline and an overview of the main composition types that I formalize.

```
case year of
1914 : SymmetricPierOcean;
1915 : AsymmetricPierOcean;
1916 : ShortBlackLines;
1917 : if random(2) = 0
then ColorPlanesShortBlackLines
else ColorPlanes;
1919 : CheckerBoard;
1920 : ColorPlanesLongBlackLines;
1922 : Peripheral;
1925 : ThreeSidedPeripheral;
1928 : TwoSidedPeripheral;
1930 : Cross;
1932 : DoubledCross;
1936 : MoreDoubledLines;
1940 : CrossingLinesFewPlanes;
1941 : Place;
1942 : CrossingColoredLines;
end
```

References and Notes

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9. Meyer Schapiro, *Mondrian—On the Humanity of Abstract Painting* (New York: George Braziller, 1995 [1982]).

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