

# Introducing Electronics

## Capacitance and Capacitors

# Capacitance and Capacitors



Figure 4.1: A capacitor.

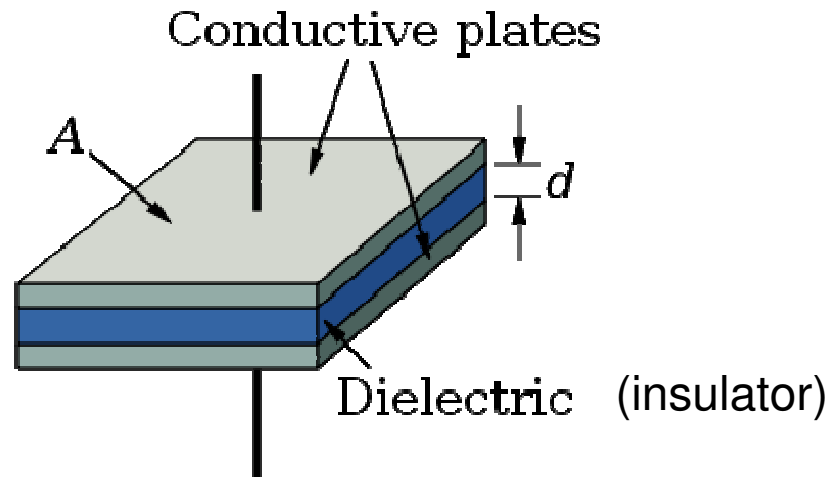
- **Capacitance** is defined as a measure of the amount of electrical charge stored for a given potential difference.
- Typical values for capacitors are in the order of pF to  $\mu\text{F}$

# Units

	<u>prefix name</u>	<u>prefix symbol</u>	<u>power-of-ten</u>
lower case prefix symbols	yocto	y	$10^{-24}$
	zepto	z	$10^{-21}$
	atto	a	$10^{-18}$
	femto	f	$10^{-15}$
	pico	p	$10^{-12}$
	nano	n	$10^{-9}$
	micro	$\mu$	$10^{-6}$
	milli	m	$10^{-3}$
	centi	c	$10^{-2}$
	deci	d	$10^{-1}$
	[unity]	[none]	$10^0$
	deka	da	$10^{+1}$
	hecto	h	$10^{+2}$
	kilo	k	$10^{+3}$
	mega	M	$10^{+6}$
upper case prefix symbols	giga	G	$10^{+9}$
	tera	T	$10^{+12}$
	peta	P	$10^{+15}$
	exa	E	$10^{+18}$
	zetta	Z	$10^{+21}$
	yotta	Y	$10^{+24}$

From [www.poynton.com/notes/units/index.html](http://www.poynton.com/notes/units/index.html)

# Capacitors



capacitance in Farads  $\rightarrow C = \epsilon_0 \epsilon_r \frac{A}{D}$

$\epsilon_0$  permittivity of space

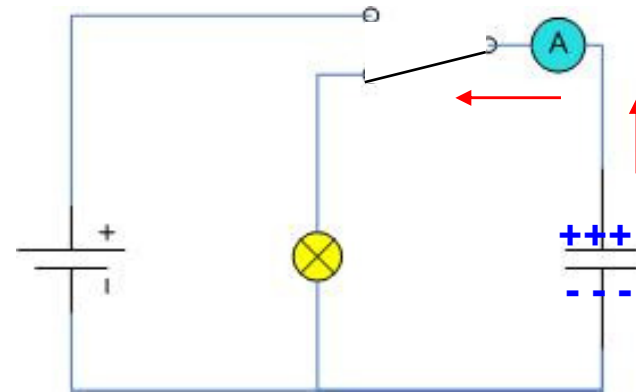
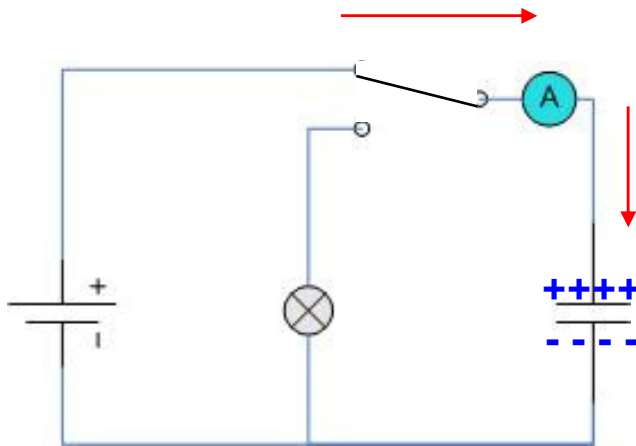
$\epsilon_r$  dielectric constant

$A$  area of the capacitor plates

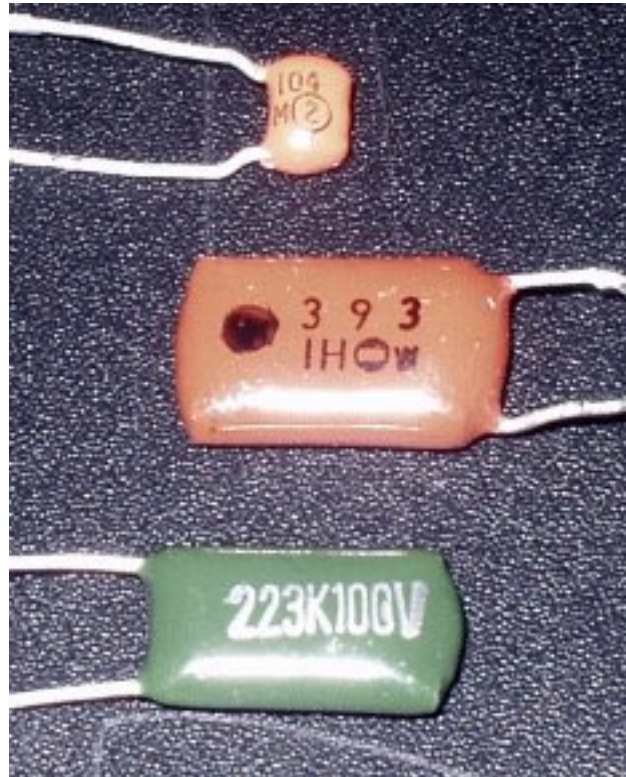
$D$  distance between the capacitor plates

# Capacitors

## Water tank – charge and discharge



# Capacitor Labelling



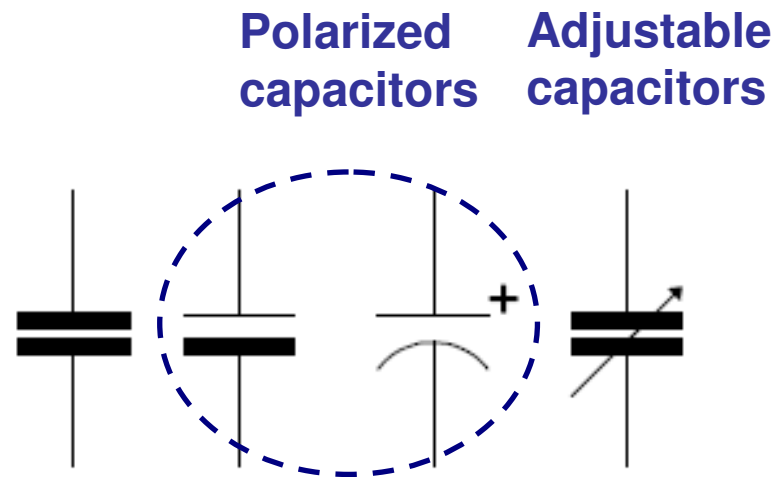
$10 \cdot 10^4 \text{ pF}$

$39 \cdot 10^3 \text{ pF}$

$22 \cdot 10^3 \text{ pF}$

K means the tolerance.

# Schematic Symbols



**Figure 4.2:** *Schematic symbols for capacitances.*

# Relation between Voltage and Current

$$C = \frac{Q}{V}. \quad (4.1)$$

$$Q = C \cdot V. \quad (4.2)$$

$$I = \frac{dQ}{dt} = C \frac{dV}{dt}. \quad (4.3)$$

- **Capacitance** is defined as a measure of the amount of electrical charge stored for a given potential difference.
- For a capacitor, the current depends on the changes in voltage.
- When the voltage over a 1 F capacitor changes with 1 V per second, there will flow a current of 1 A.

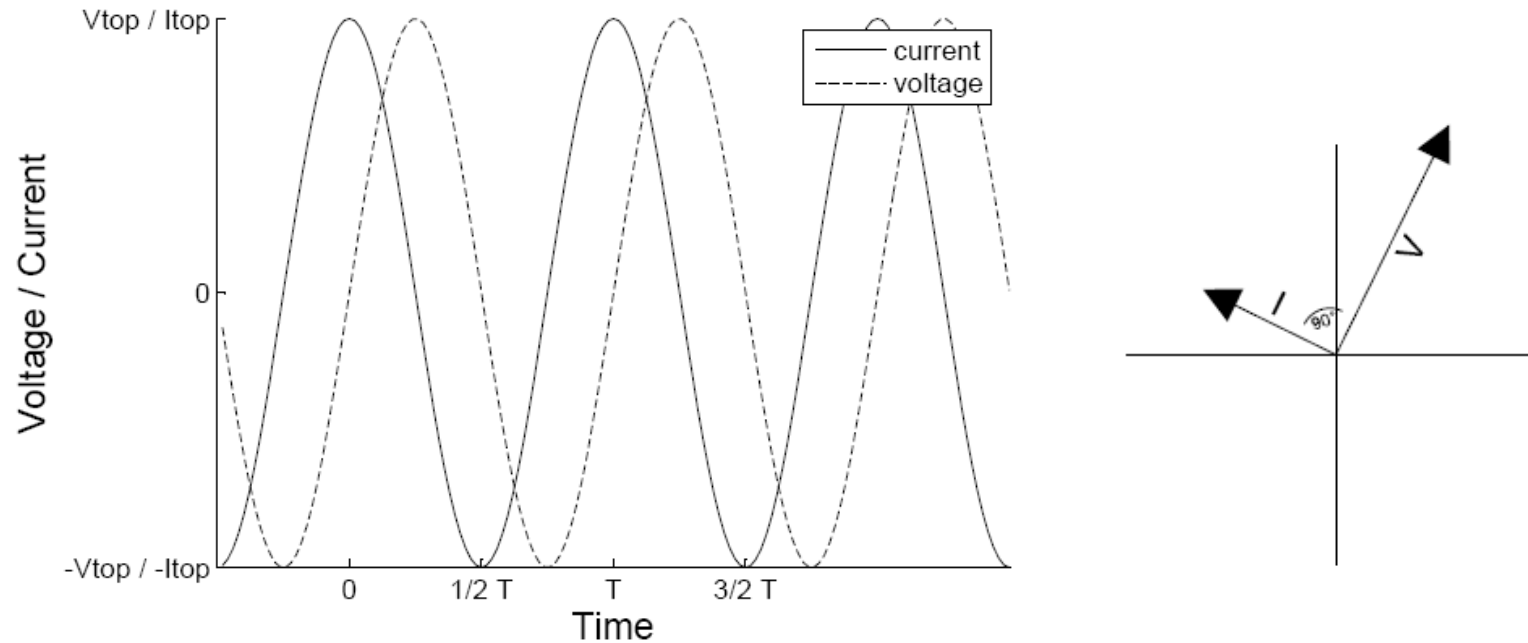


# Electrical Energy

$$E = \frac{1}{2}CV^2. \quad (4.4)$$

- The stored charge results in an electrical energy E
- The unit of energy is Joule.

# Frequency Dependence

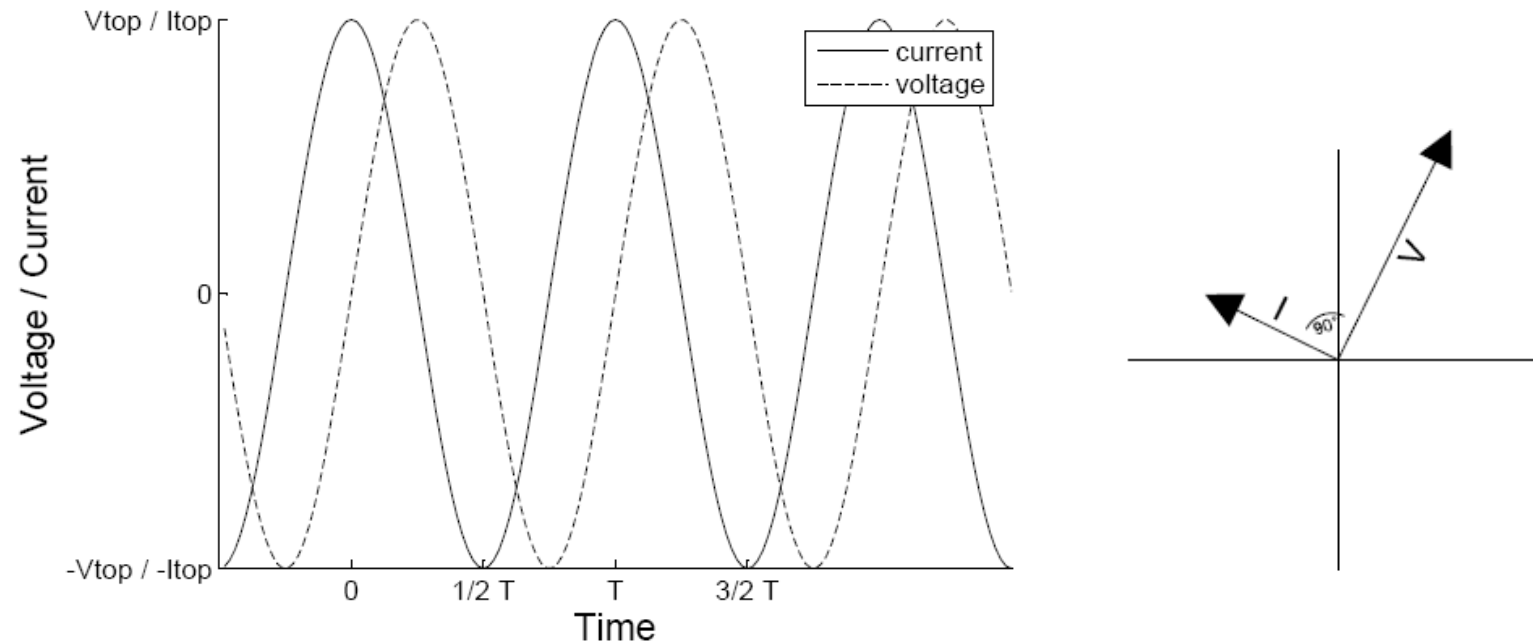


**Figure 4.4:** For a capacitor, the current is  $90^\circ$  ahead of the voltage.

$$I = \frac{dQ}{dt} = C \frac{dV}{dt}. \quad (4.3)$$

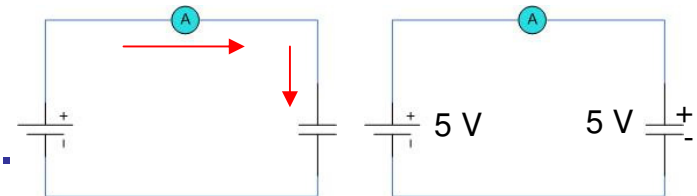
$$V = \sin(\omega t), I = C \omega \cos(\omega t) = C \omega \sin(\omega t + 90^\circ)$$

# Frequency Dependence



**Figure 4.4:** For a capacitor, the current is  $90^\circ$  ahead of the voltage.

- At maximum voltage, no current.
- When voltage changes fastest, current is maximum.



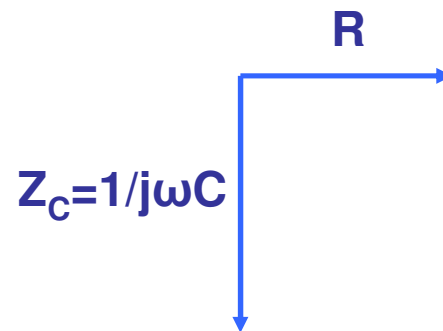
# Frequency Dependence

$$Z_c(\omega) = \frac{1}{\omega C} = \frac{1}{2\pi f C}. \quad (4.5)$$



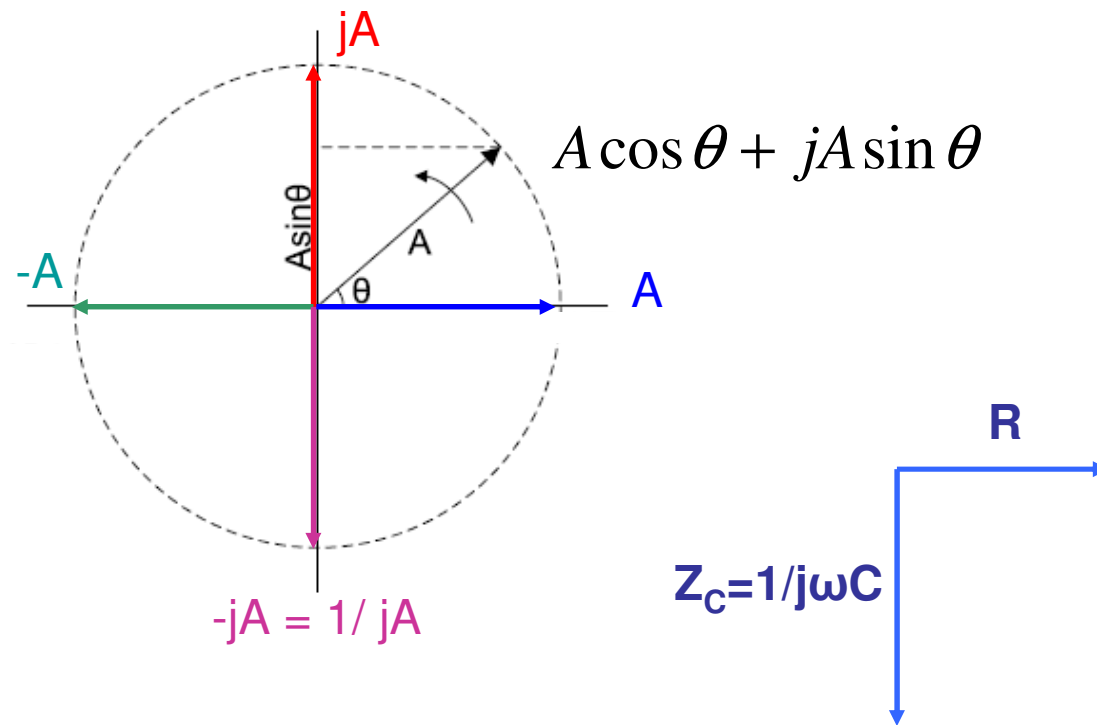
## Note

In literature you may find a more complex form for the impedance of a capacitor. However, for this assignment the relation given in Equation 4.5 is sufficient, because we do not want to introduce the required complex mathematics.

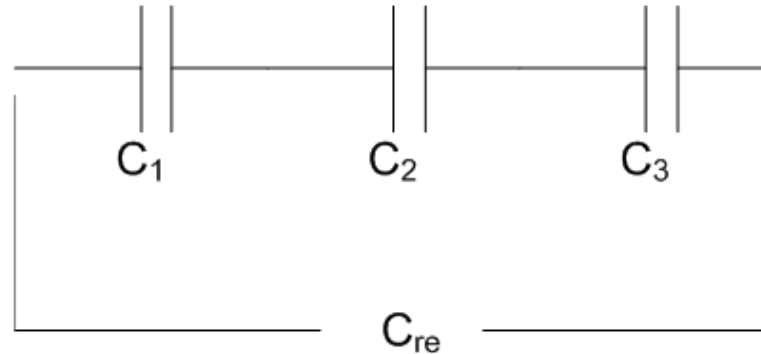


# Complex Numbers

$$\sqrt{-1} = ?j$$



# Series Connection

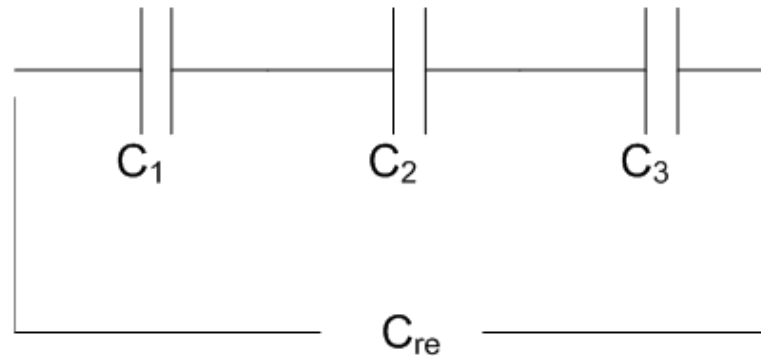


**Figure 4.5:** *Example of a series connection of three capacitors.*

Like the situation in series connected resistors

- The current through all capacitors is the same.
- Voltage drop across the capacitors equals to the sum of voltage drop across each capacitor.

# Series Connection



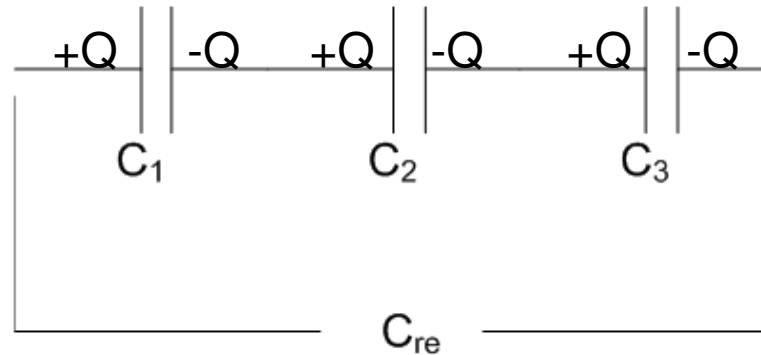
**Figure 4.5:** *Example of a series connection of three capacitors.*

Unlike the situation in series connected resistors

$$C_{re} = \frac{1}{\sum_{i=1}^N \frac{1}{C_i}}. \quad (4.6)$$

$$C = \epsilon_0 \epsilon_r \frac{A}{D} \quad \text{the same area, longer distance}$$

# Series Connection



**Figure 4.5:** *Example of a series connection of three capacitors.*

$$Q = C_1 \cdot V_1 = C_2 \cdot V_2 = C_3 \cdot V_3 = C_{re}(V_1 + V_2 + V_3)$$

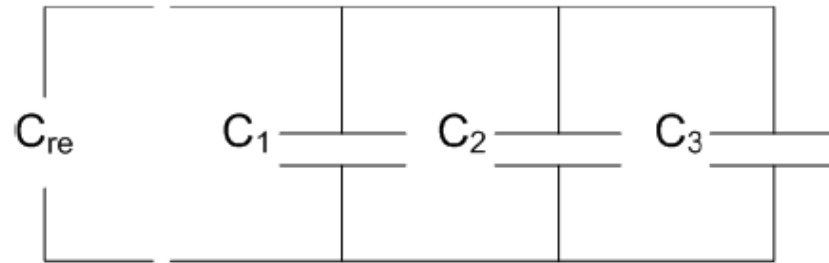
$$\left(\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}\right)C_{re} = Q$$

$$\frac{1}{C_{re}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_{re} = \frac{1}{\sum_{i=1}^N \frac{1}{C_i}}$$



# Parallel Connection

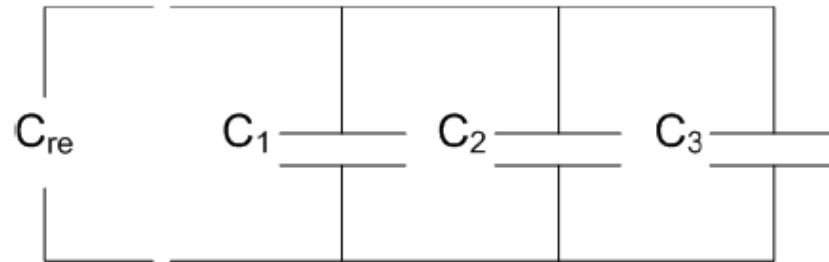


**Figure 4.6:** *Example of a parallel connection of three capacitors.*

Like the situation in parallel connected resistors

- The voltage drop across all capacitors is the same.
- The current through capacitors equals to the sum of current through each capacitor.

# Parallel Connection



**Figure 4.6:** *Example of a parallel connection of three capacitors.*

Unlike the situation in parallel connected resistors

$$C_{re} = \sum_{i=1}^N C_i. \quad (4.7)$$

$$C = \epsilon_0 \epsilon_r \frac{A}{D} \quad \text{the same distance, larger area}$$

# Capacitors and Filters

- In many applications it is necessary to filter out frequency content.
- Low pass filters, High pass filters, Band pass filters

# Why Filters?

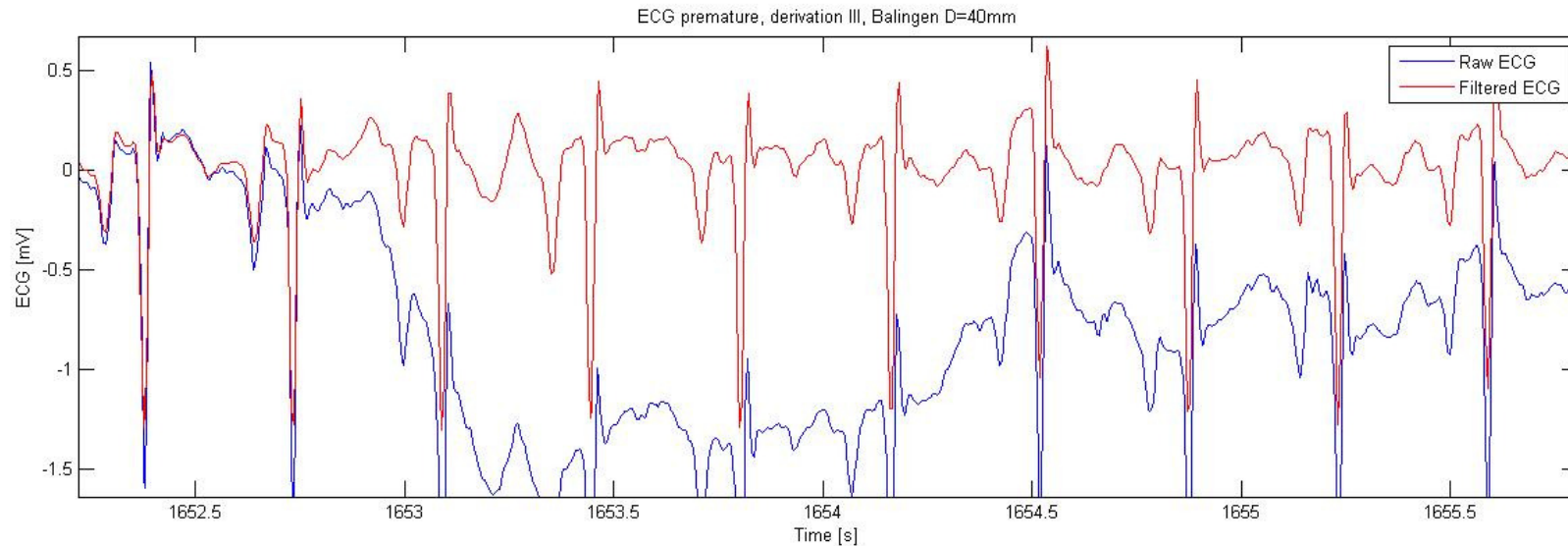
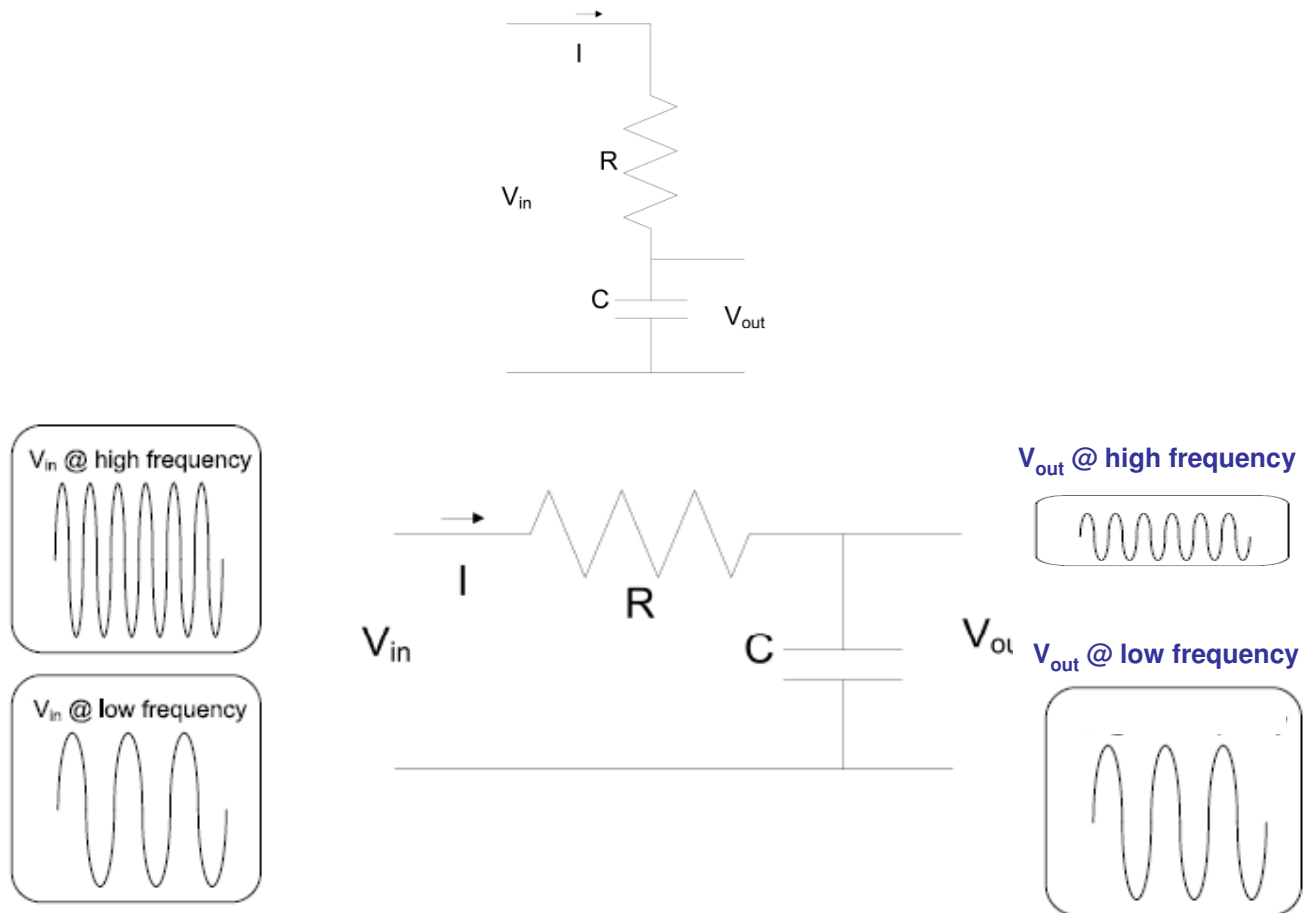


Figure from Master thesis, Sibrecht Bouwstra, 2008

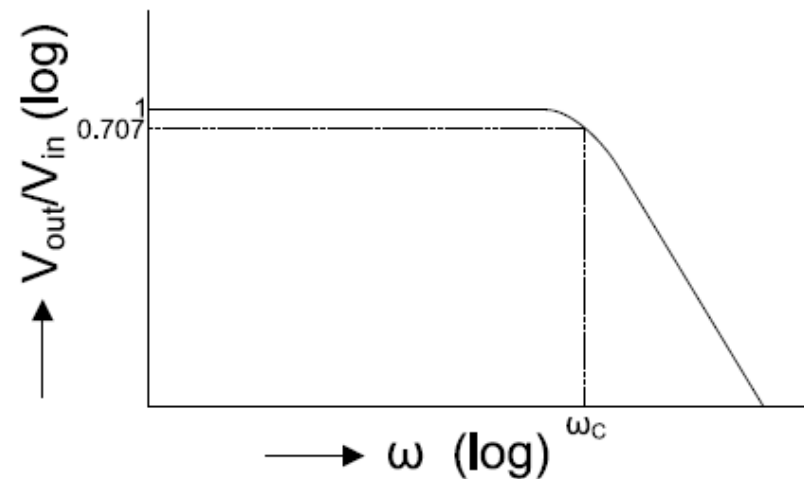
# RC Low Pass Filters



# RC Low Pass Filters

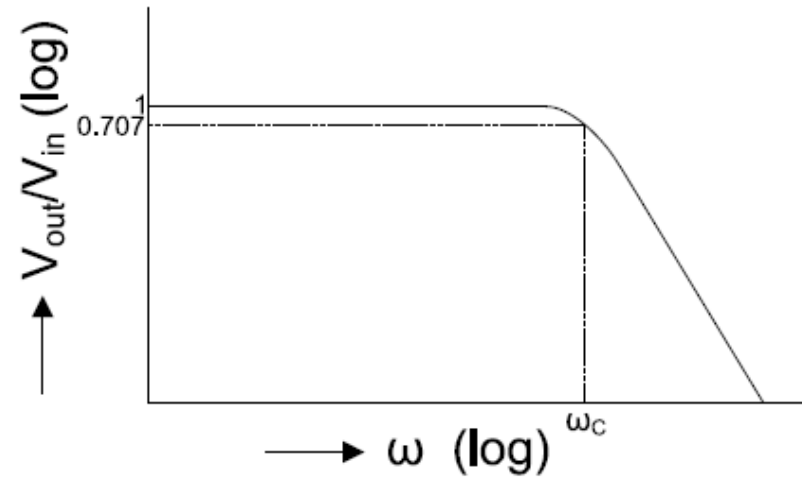
$$X_{dB} = 10 \log_{10} \frac{P_1}{P_0}. \quad (4.9)$$

$$X_{dB} = 20 \log_{10} \frac{V_1}{V_0}. \quad (4.10)$$



**Figure 4.9:** *Amplitude transfer-function of an low-pass RC-filter.*

# RC Low Pass Filters



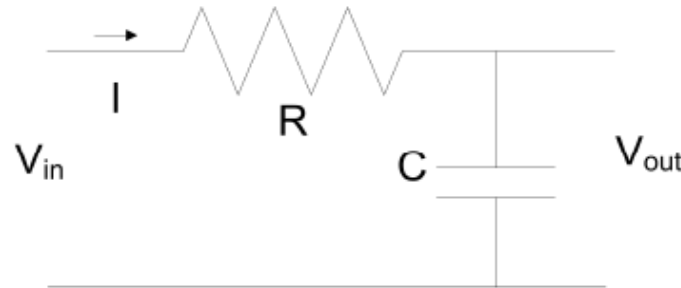
**Figure 4.9:** Amplitude transfer-function of an low-pass  $RC$ -filter.

At the cut-off frequency,  $R = Z_c = 1/\omega C$

$$\omega_{(cut-off)} = \frac{1}{RC} \text{ or } : f_{-3dB} = \frac{1}{2\pi RC}. \quad (4.11)$$

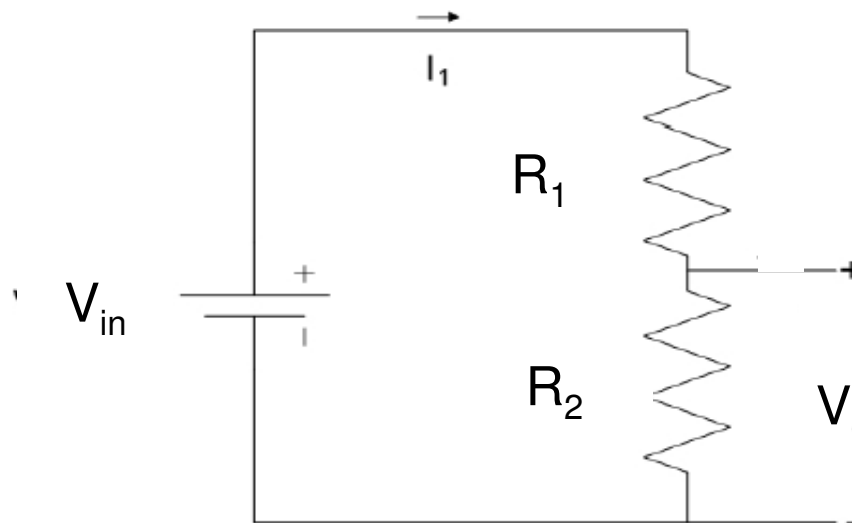
- At the cut-off frequency (-3 dB), the output power is half of the input power.
- Filters, especially RC-filters, are specified by its -3 dB frequency.

# RC Low Pass Filters



$$V_{out} = \frac{Z_c}{R + Z_c} V_{in}$$

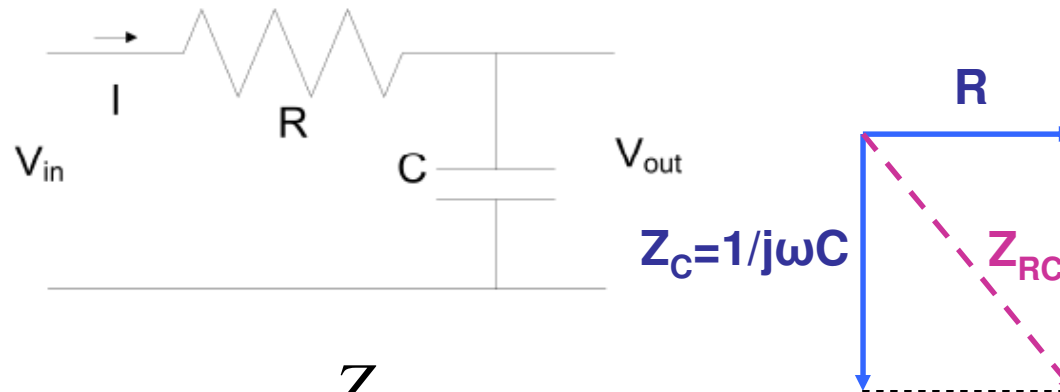
Voltage divider



$$V_{out} = V_{in} R_2 / (R_1 + R_2)$$



# RC Low Pass Filters



$$V_{out} = \frac{Z_c}{R + Z_c} V_{in}$$

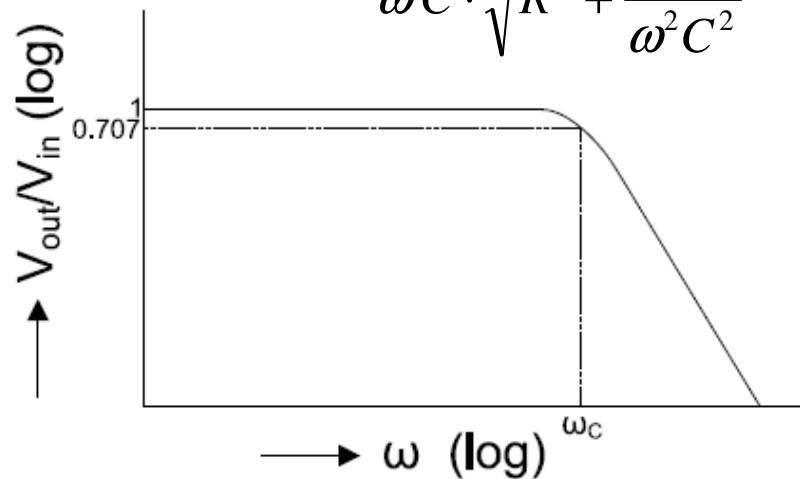
Not a simple “add” operation !

$$\left| V_{out}(\omega) \right| = \frac{Z_c}{Z_{re}} \cdot V_{in}(\omega) = \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cdot \left| V_{in}(\omega) \right|. \quad (4.8)$$

# RC Low Pass Filters

$$\left| V_{out}(\omega) \right| = \frac{Z_c}{Z_{re}} \cdot V_{in}(\omega) = \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cdot \left| V_{in}(\omega) \right|. \quad (4.8)$$

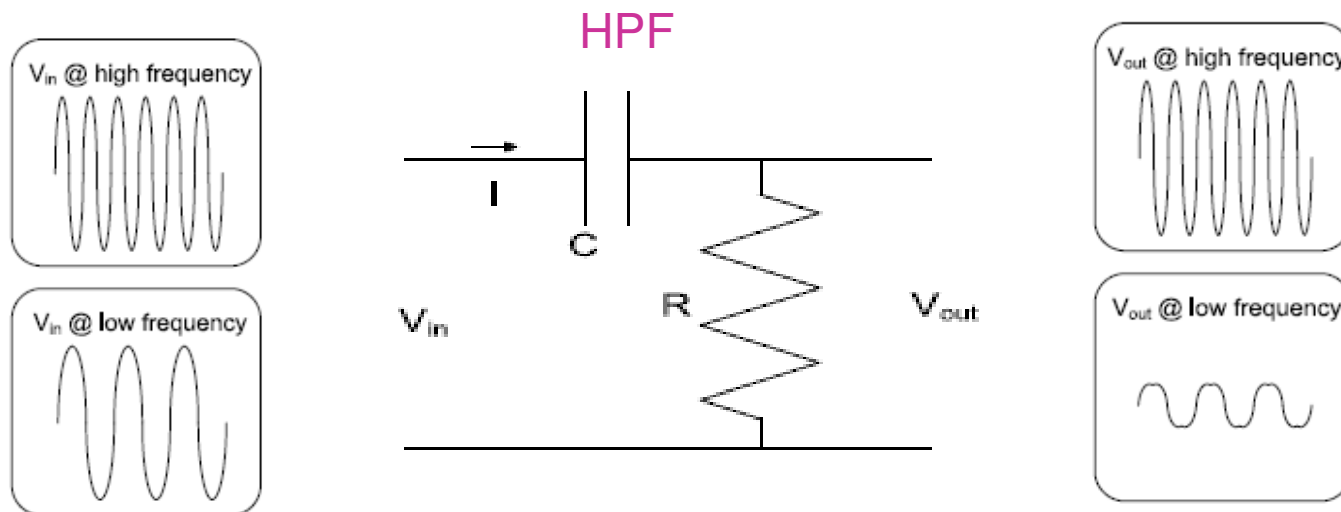
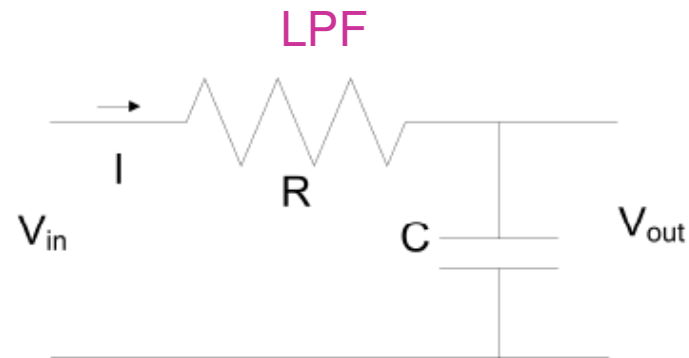
$$= \frac{\omega C \cdot \frac{1}{\omega C}}{\omega C \cdot \sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \left| V_{in}(\omega) \right| = \frac{1}{\sqrt{R^2 \omega^2 C^2 + 1}} \left| V_{in}(\omega) \right|$$



- $\omega = 0, |V_{out}| = |V_{in}|$
- $\omega = \infty, |V_{out}| = 0$
- $\omega = \omega_c, |V_{out}| = 0.707|V_{in}|$

Figure 4.9: Amplitude transfer-function of an low-pass RC-filter.

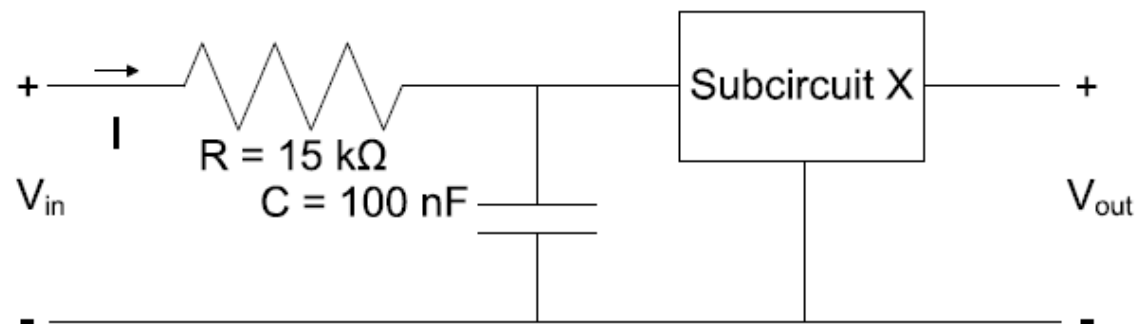
# RC High Pass Filters



# Capacitors and Delay-Units

- Charging a capacitor takes time.
- You can use capacitors to generate a delay.

# Capacitors and Delay-Units

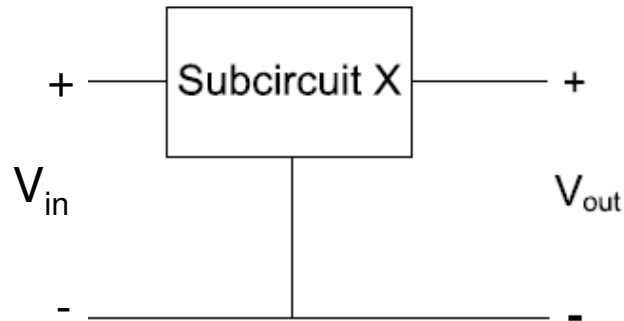


**Figure 4.11:** *Schematic of a delay-unit and a subcircuit.*

Function of the subcircuit X:

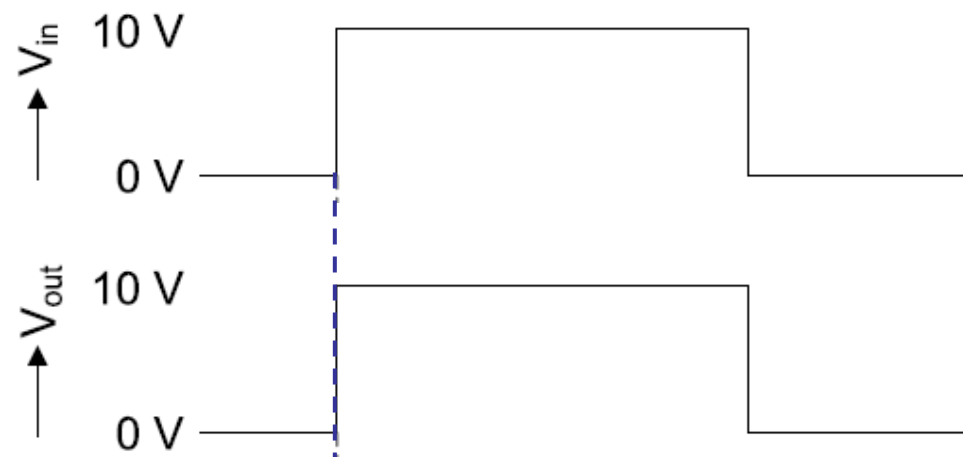
- When  $V_c$  is between 5 V and 10 V,  $V_{out} = 10\text{ V}$ .
- When  $V_c$  is equal or less than 5 V,  $V_{out} = 0\text{ V}$ .

# Capacitors and Delay-Units



Function of the subcircuit X:

- When  $V_{in}$  is between 5 V and 10 V,  $V_{out} = 10$  V.
- When  $V_{in}$  is equal or less than 5 V,  $V_{out} = 0$  V.



# Capacitors and Delay-Units

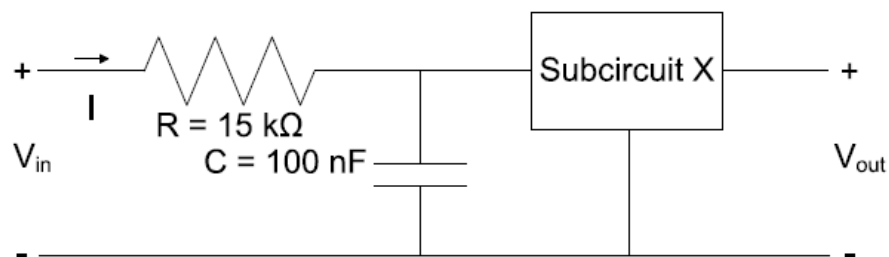
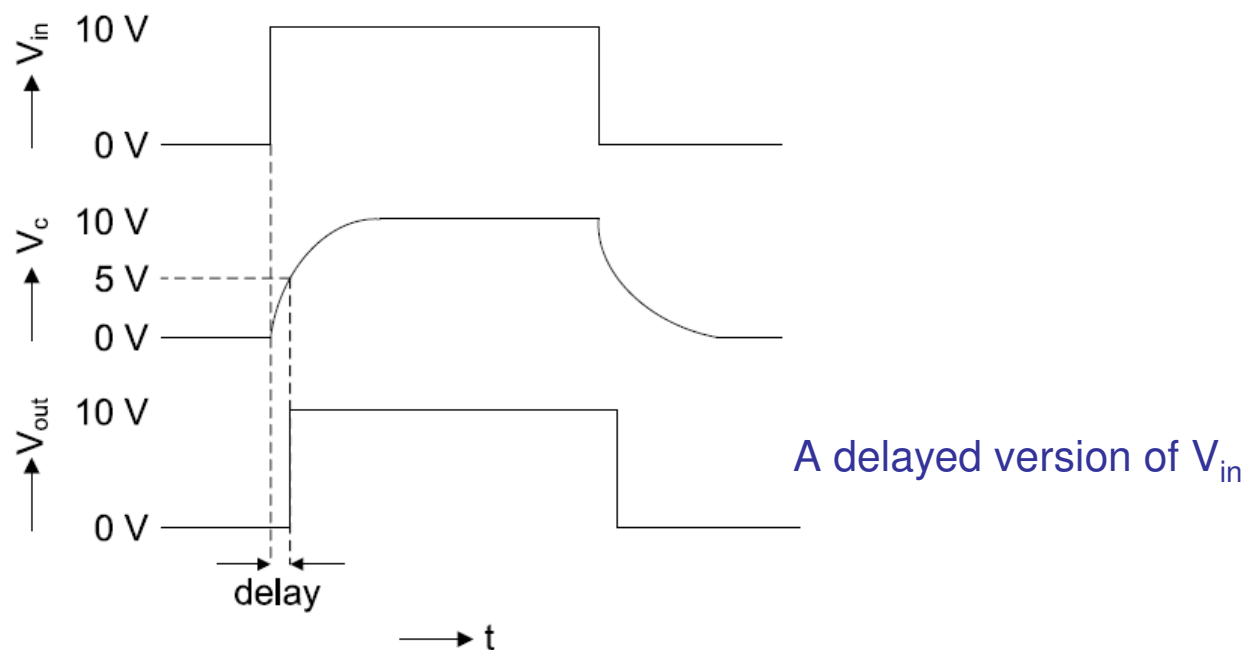


Figure 4.11: Schematic of a delay-unit and a subcircuit.

Function of the subcircuit X:

- When  $V_{in}$  is between 5 V and 10 V,  $V_{out} = 10$  V.
- When  $V_{in}$  is equal or less than 5 V,  $V_{out} = 0$  V.



# Introducing Electronics

## Week 3

### Inductance and Inductors



# Inductance and Inductors

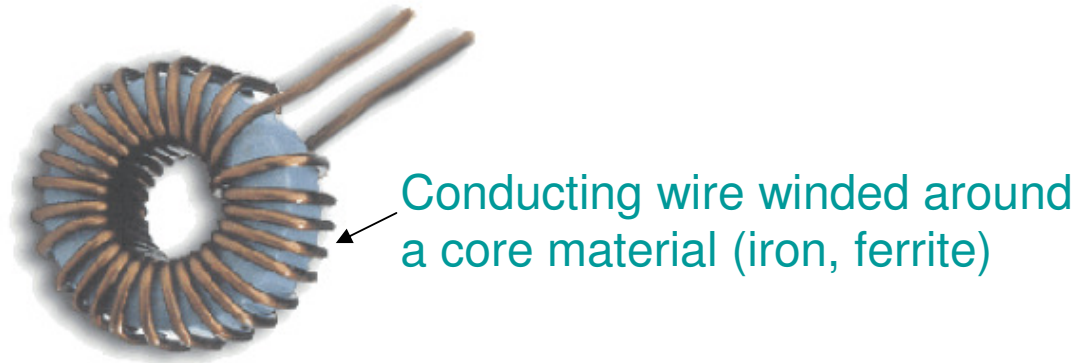


Figure 5.1: *An inductor.*

- **Inductance** is defined as a measure of the amount of magnetic flux produced for a given electric current.
- Typical values for inductors are in the order of  $\mu\text{H}$  to  $\text{mH}$ .

# Inductance and Inductors

- DC resistance of an inductor comes from the non-zero resistance of the wire used in windings.
- The core often has a shape of a bar or a ring.



**Figure 5.2:** *Schematic symbol for inductance.*

# Relation between Voltage and Current

$$L = \frac{\Phi}{I}. \quad (5.1)$$

$$\Phi = L \cdot I. \quad (5.2)$$

$$V = \frac{d\Phi}{dt} = L \frac{dI}{dt}. \quad (5.3)$$

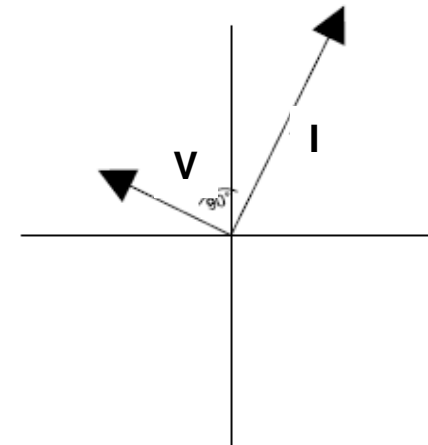
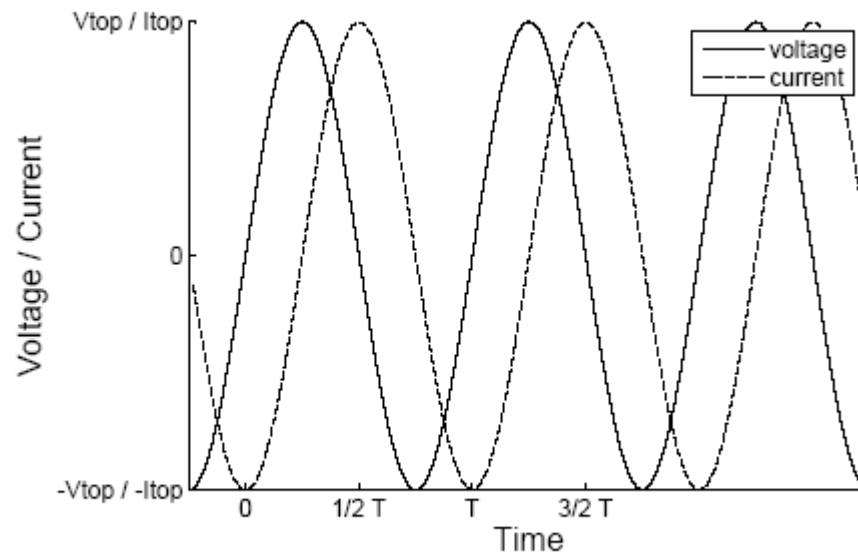
- For an inductor, the voltage depends on the changes in current.
- When the current over a 1 H inductor changes with 1 A per second, there will be a voltage of 1 V.

# Magnetic Energy

$$E_{\text{magnetic}} = \frac{1}{2} \cdot LI^2 \quad (5.4)$$

- Magnetic energy  $E_{\text{magnetic}}$  is stored.
- The unit of energy is Joule.

# Power and Frequency Behavior

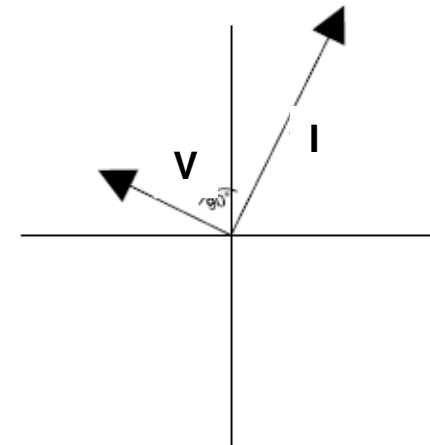
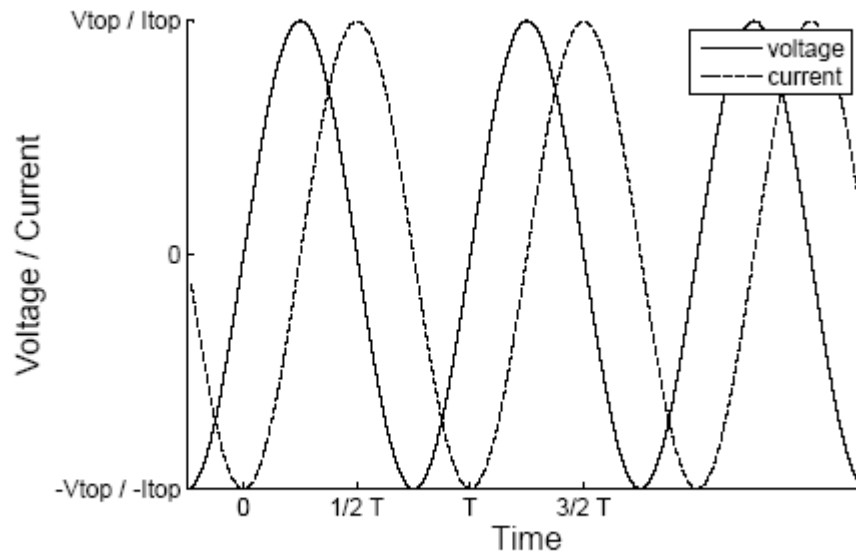


**Figure 5.3:** For an inductor, the voltage is  $90^\circ$  ahead of the current.

$$V = \frac{d\Phi}{dt} = L \frac{dI}{dt}. \quad (5.3)$$

$$I = \sin(\omega t), \quad V = L \omega \cos(\omega t) = L \omega \sin(\omega t + 90^\circ)$$

# Power and Frequency Behavior



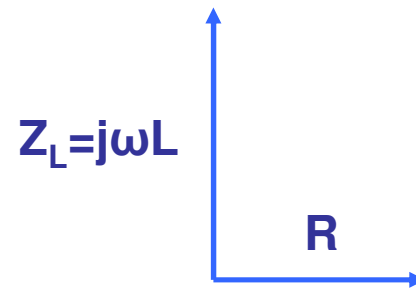
**Figure 5.3:** For an inductor, the voltage is  $90^\circ$  ahead of the current.

- At maximum current, no voltage.
- When current changes fastest, voltage is maximum.

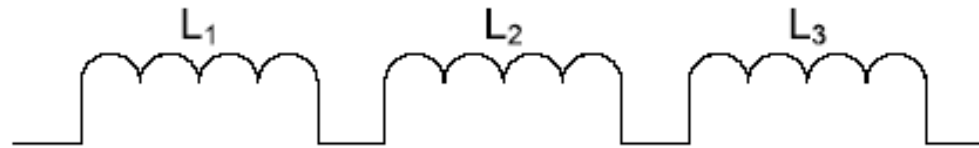
# Power and Frequency Behavior

$$Z_l(\omega) = \omega L = 2\pi f L. \quad (5.5)$$

Equation 5.5 does not take into account the phase shift.



# Series Connection



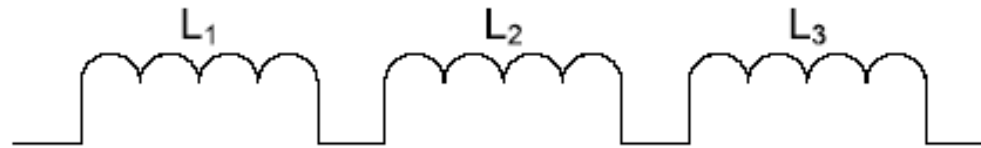
**Figure 5.4:** *Example of a series connection of three inductors.*

Like the situation in series connected resistors

- The current through all inductors is the same.
- Voltage drop across the inductors equals to the sum of voltage drop across each inductor.



# Series Connection

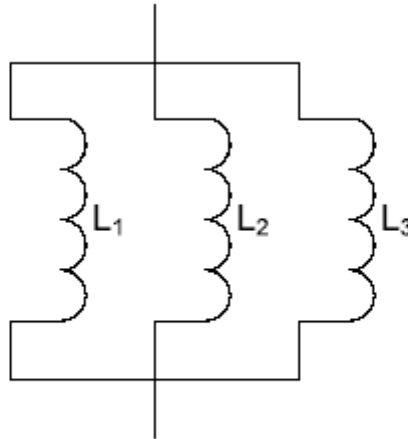


**Figure 5.4:** *Example of a series connection of three inductors.*

Similar formula as for series connected resistors

$$L_{re} = \sum_{i=1}^N L_i. \quad (5.6)$$

# Parallel Connection

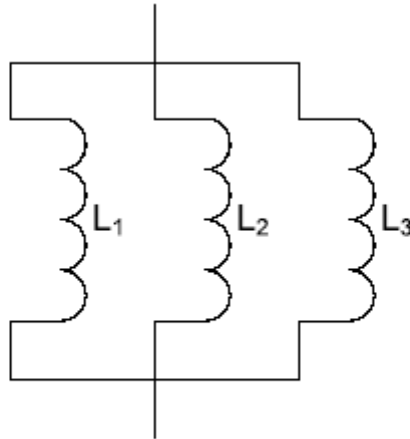


**Figure 5.5:** *Example of a parallel connection of three inductors.*

Like the situation in parallel connected resistors

- The voltage drop across all inductors is the same.
- The current through all inductors equals to the sum of current through each inductor.

# Parallel Connection



**Figure 5.5:** *Example of a parallel connection of three inductors.*

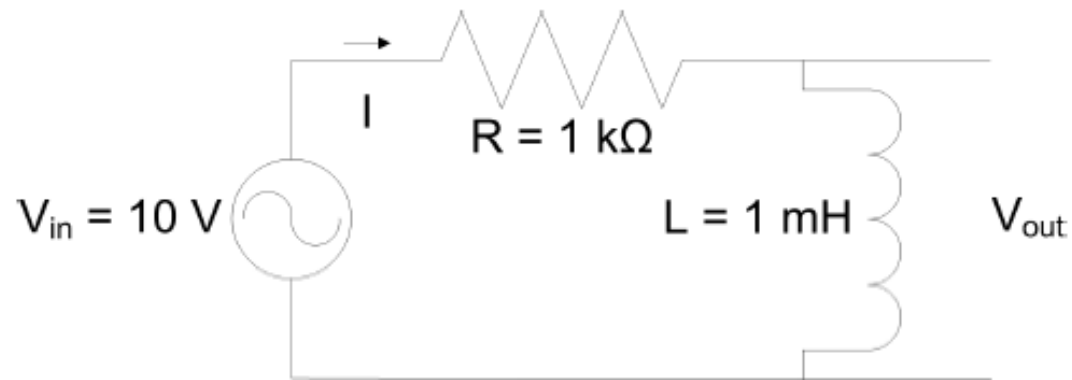
Similar formula as for parallel connected resistors

$$L_{re} = \frac{1}{\sum_{i=1}^N \frac{1}{L_i}}. \quad (5.7)$$

# Inductors and Filters

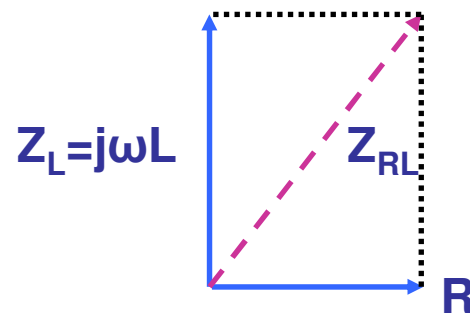
- In many applications it is necessary to filter out frequency content.
- Low pass filters, High pass filters, Band pass filters

# RL Filters

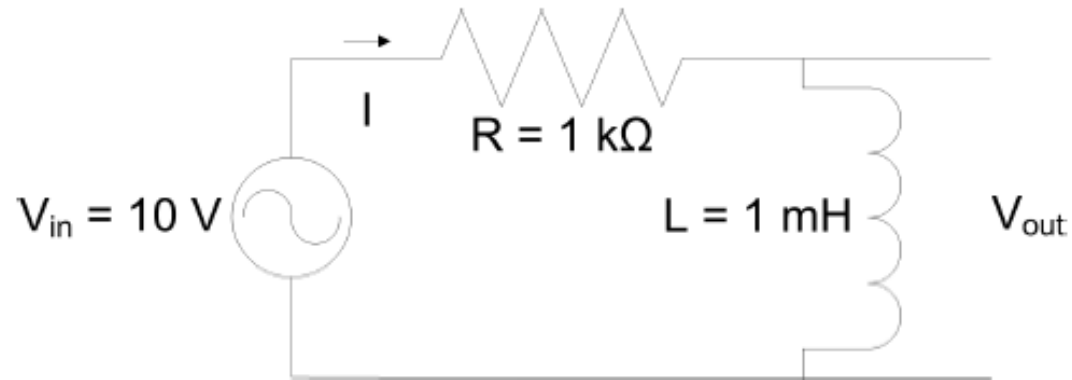


**Figure 5.6:** Schematic of a simple *RL*-filter.

$$\left| V_{out}(\omega) \right| = \frac{Z_L}{Z_{re}} \cdot \left| V_{in}(\omega) \right| = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \left| V_{in}(\omega) \right| \quad (5.8)$$



# RL Filters



**Figure 5.6:** *Schematic of a simple RL-filter.*

$$\left| V_{out}(\omega) \right| = \frac{Z_L}{Z_{re}} \cdot \left| V_{in}(\omega) \right| = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \left| V_{in}(\omega) \right| \quad (5.8)$$

**LPF or HPF?**

# RL Filters

- At the cut-off frequency (-3 dB), the output power is half of the input power.

$$\left| V_{out}(\omega) \right| = \frac{Z_L}{Z_{re}} \cdot \left| V_{in}(\omega) \right| = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \left| V_{in}(\omega) \right| \quad (5.8)$$

At the cut-off frequency,  $R = Z_L = \omega L$

$$\omega_{(cut-off)} = \frac{R}{L} \text{ or } : f_{-3dB} = \frac{R}{2\pi L}. \quad (5.9)$$

# RL Filters

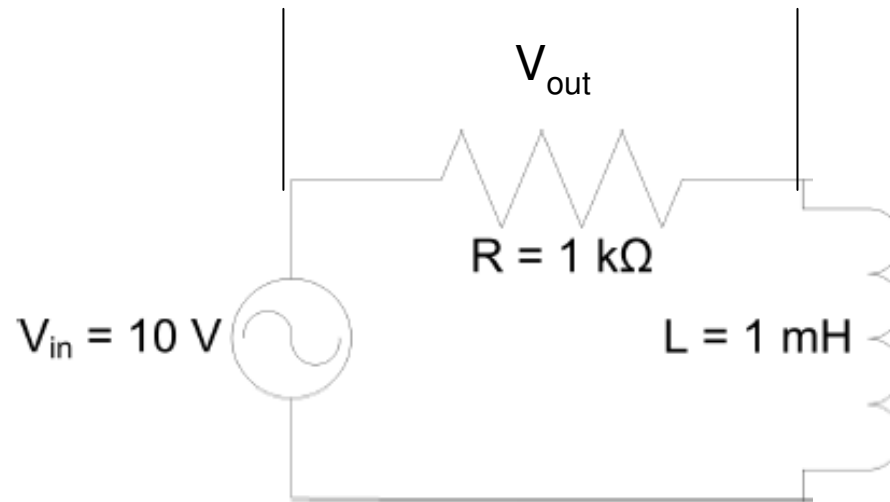


Figure 5.6: Schematic of a simple  $RL$ -filter.

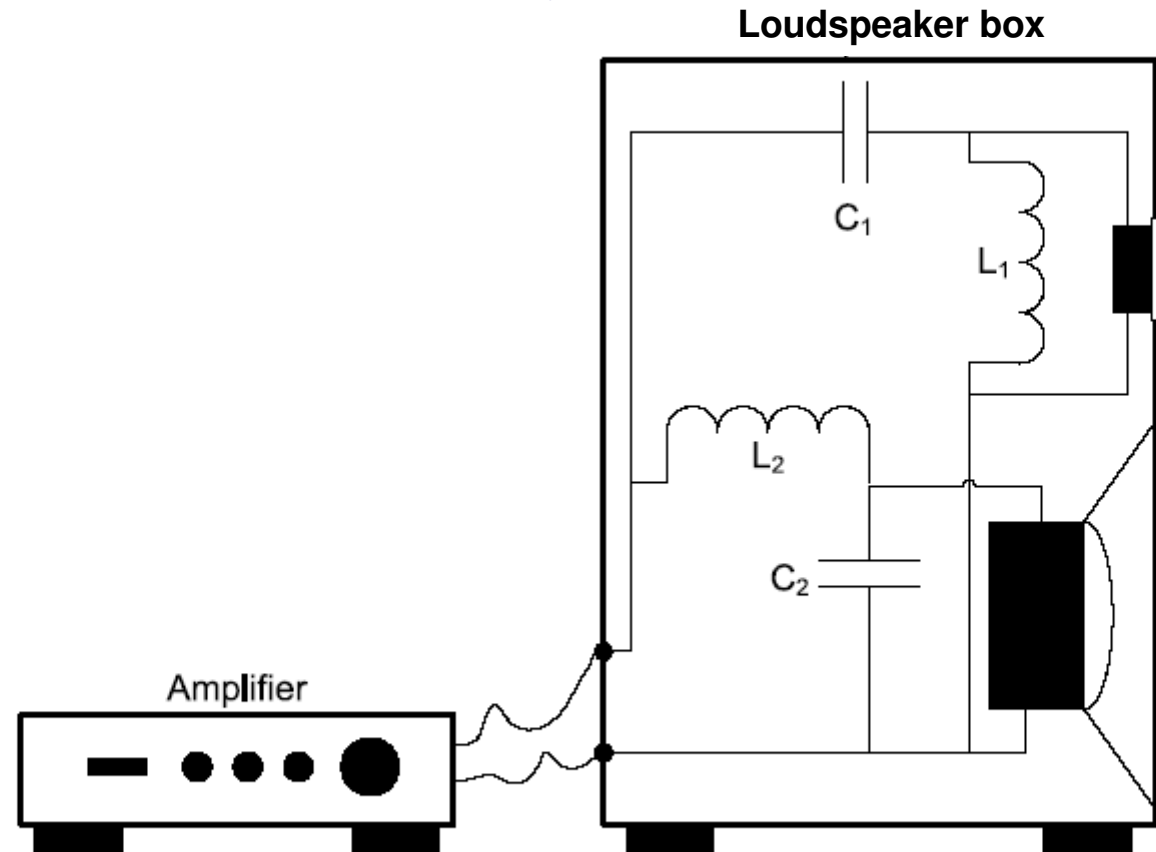
$$\left| V_{out}(\omega) \right| = \frac{R}{Z_{re}} \cdot \left| V_{in}(\omega) \right| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \left| V_{in}(\omega) \right| \quad (5.8)$$

**LPF or HPF?**



# Filters

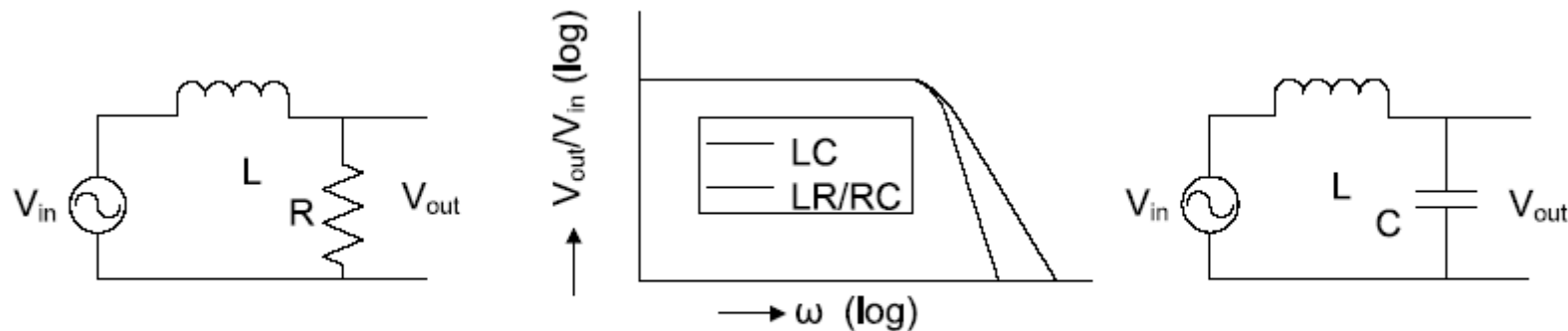
Besides RC/RL filters, LC and RLC filters exist.



**Figure 5.7:** A loudspeaker box with two LC-filters. One filter is used to pass high frequencies and the other is used to pass low frequencies.

# Filters

- Besides RC/RL filters, LC and RLC filters exist.
- LC/CL filters contain a frequency square in the denominator of their amplitude transfer function.
- Amplitude transfer function decreases/increases much faster for LC/CL filters than for RC/RL filters.



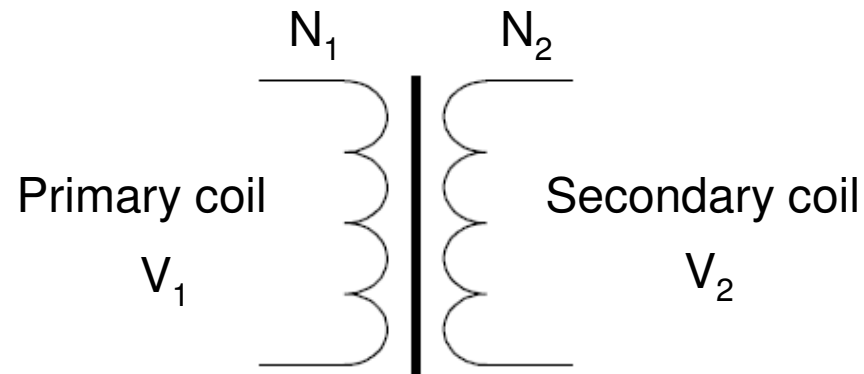
**Figure 5.8:** *The (amplitude) transfer of a LC/CL filter changes faster (is steeper) than a LR/RL/RC/CR filter.*

# Inductors and Mechanics

- Inductors are often used to transform mechanical energy to electrical energy (generator), and vice versa (motor).
- Another application of inductors is relay.
- A **relay** is an electrical switch that opens and closes under the control of another electrical circuit.
- A relay is able to control an output circuit of higher power than the input circuit.
- A relay can perform distant switch (elevator control).

<http://www.youtube.com/watch?v=CYDw-R9YYHU>

# Inductors and Transformers



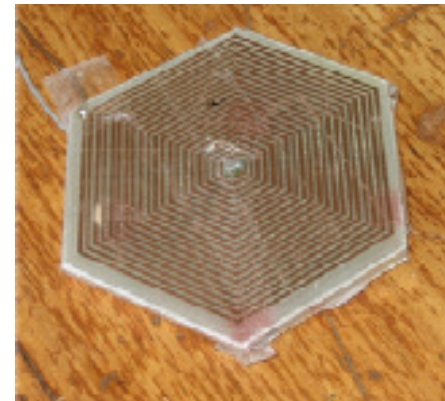
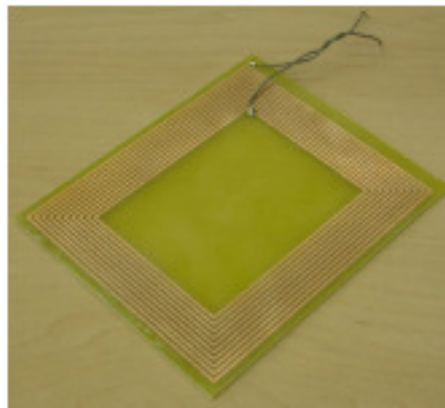
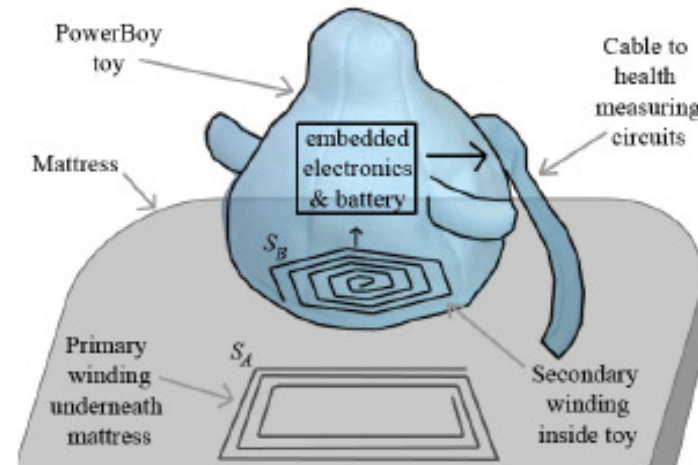
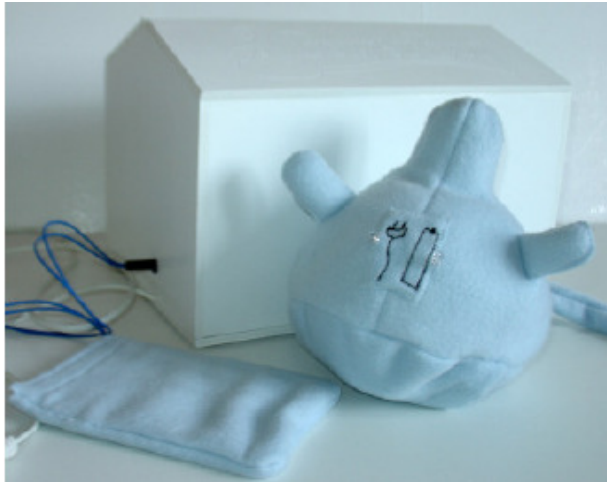
$$V_2 = V_1 \cdot N_2 / N_1$$

$$I_2 = I_1 \cdot N_1 / N_2$$

Figure 5.10: *Schematic of a transformer.*

- In power line for long distance transmission, transforming electrical power to a high voltage. Reduce line loss by lower the line current.
- Transform 230 V to a useful voltage that a machine can use.

# Example Design Project



W. Chen, C. L. W. Sonntag, F. Boesten, S. Bambang Oetomo, and L. M. G. Feijs, “A Power Supply Design of Body Sensor Networks for Health Monitoring of Neonates”, Presented at ISSNIP 2008, Sydney, Australia

