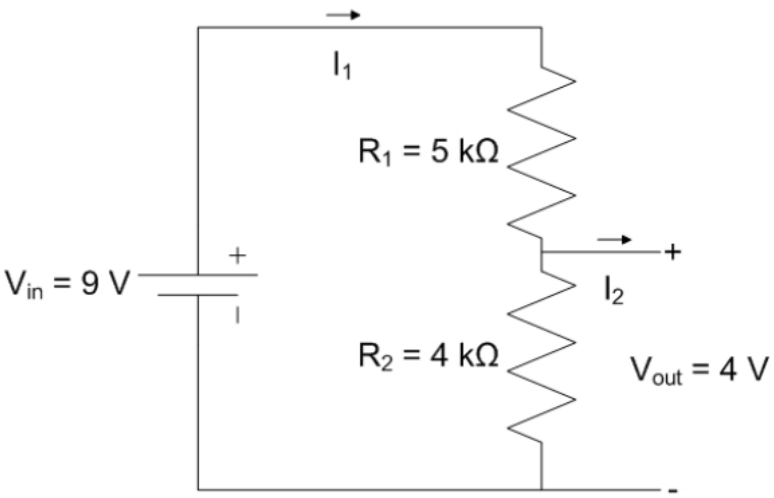
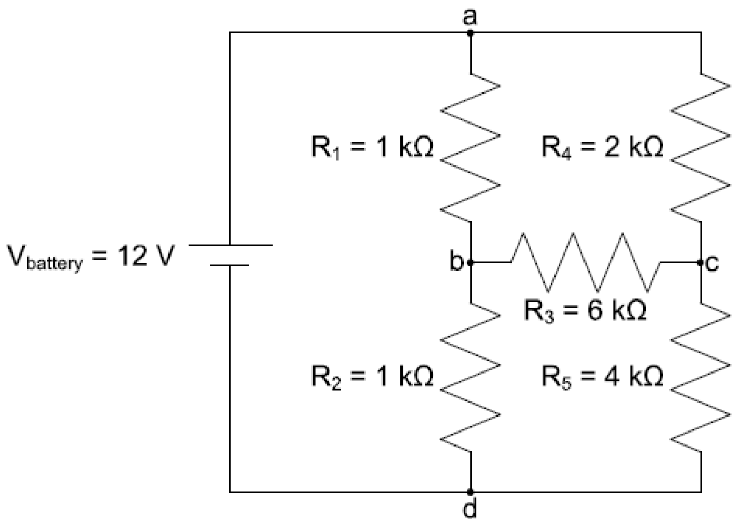
Introducing Electronics

Chapter 6 & 7
Kirchoff's Law
Theorems of Norton / Thevenin

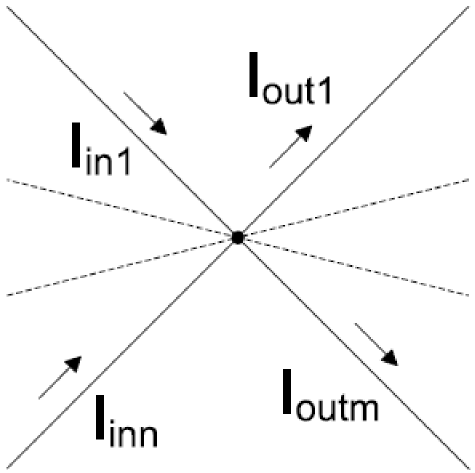
Why Kirchoff’s Law





- Ohm’s law can not deal with complex electrical circuits.
- Kirchoff’s current and voltage laws can help us analyzing complex circuits consisting of resistors, capacitors and inductors.
- Is Kirchoff’s law difficult? Logic thinking & Basic calculation

Kirchoff's Current Law - KCL



• KCL - The sum of currents entering a node is equal to the sum of currents leaving a node.

• Current cannot accumulate in a node: what goes in must come out.

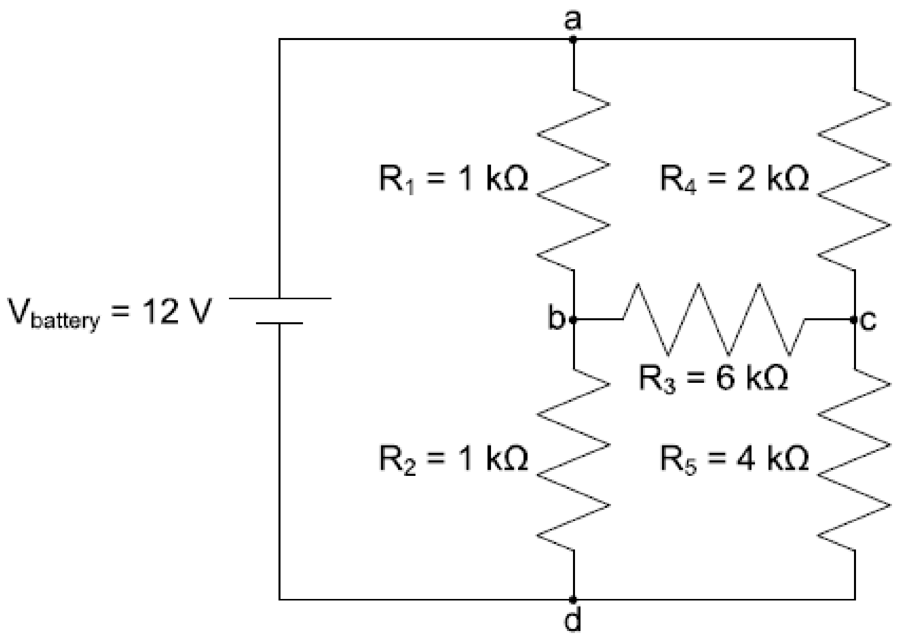
Examples - Canals, Marbles in tubes, etc.

$$I_{in1} + I_{in2} + ... + I_{inn} = I_{out1} + I_{out2} + ... + I_{outm}$$

$$\sum I_{incoming} - \sum I_{outgoing} = 0. \qquad (6.1)$$

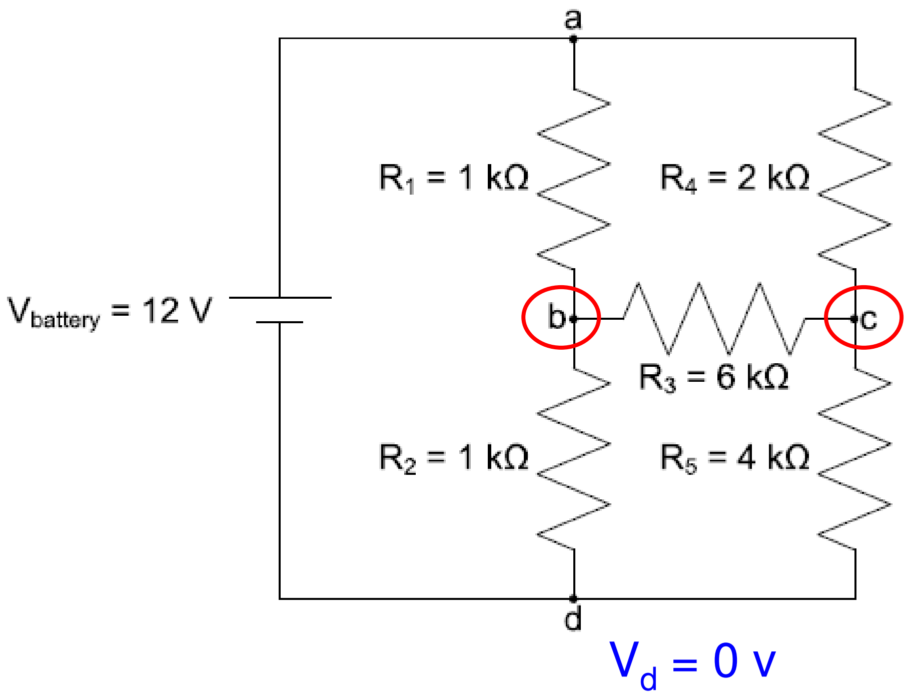
Example - Applying KCL to a Circuit

How to find all the currents and voltages in the circuit below using KCL?



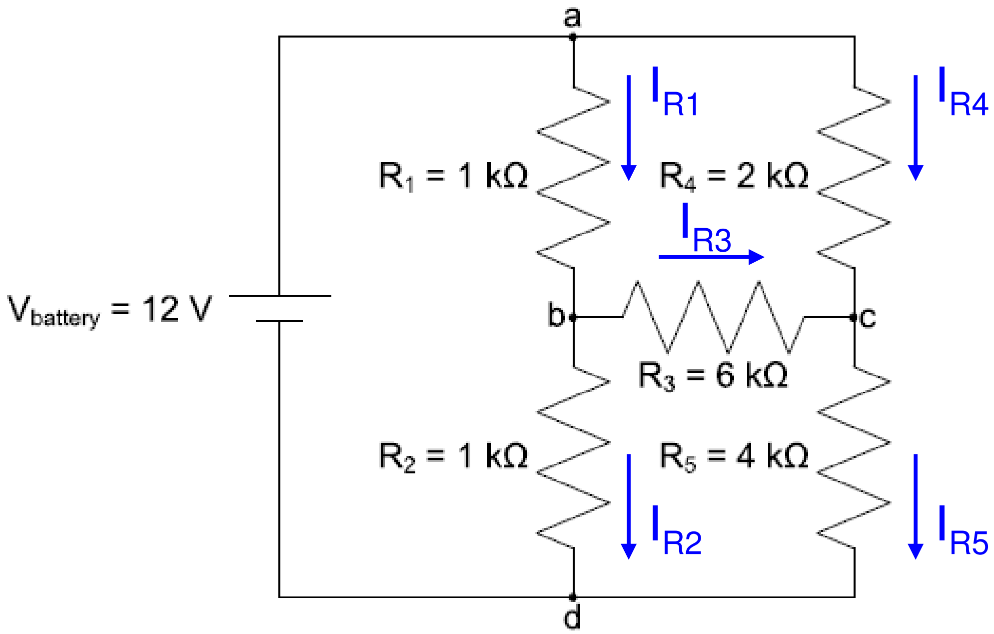
Example - Applying KCL to a Circuit

Four nodes; reference node d; b & c are special nodes for analysis

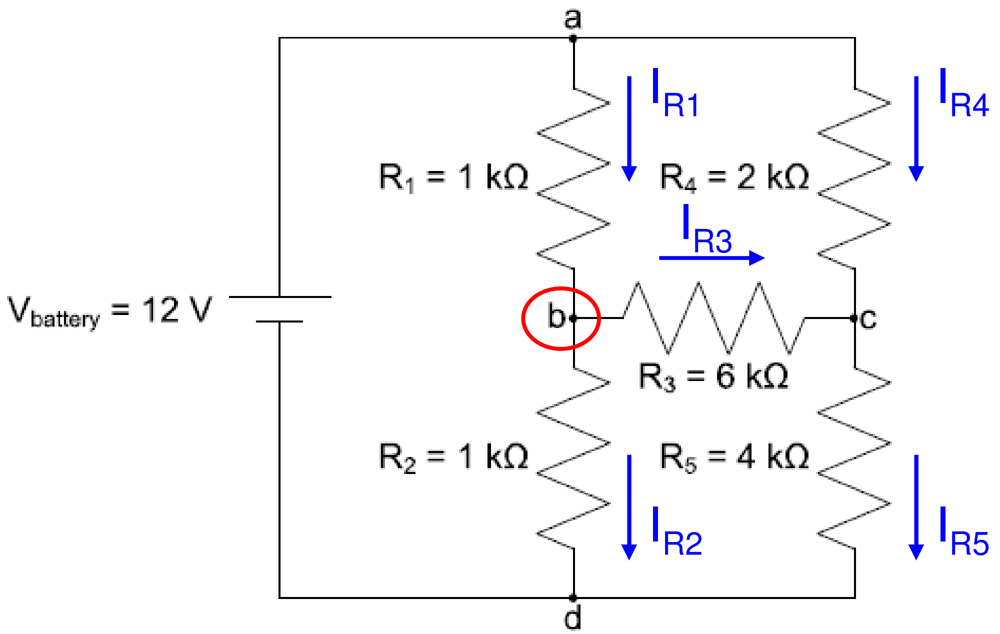


Example - Applying KCL to a Circuit

Mark the currents with direction.
Choosing the wrong direction is not a big issue.



Example - Applying KCL to a Circuit



KCL, node b

$I_{R1} = I_{R2} + I_{R3}$

Ohm's Law

$$I_{R_1} = \frac{V_a - V_b}{R_1} = \frac{12 - V_b}{1000}$$

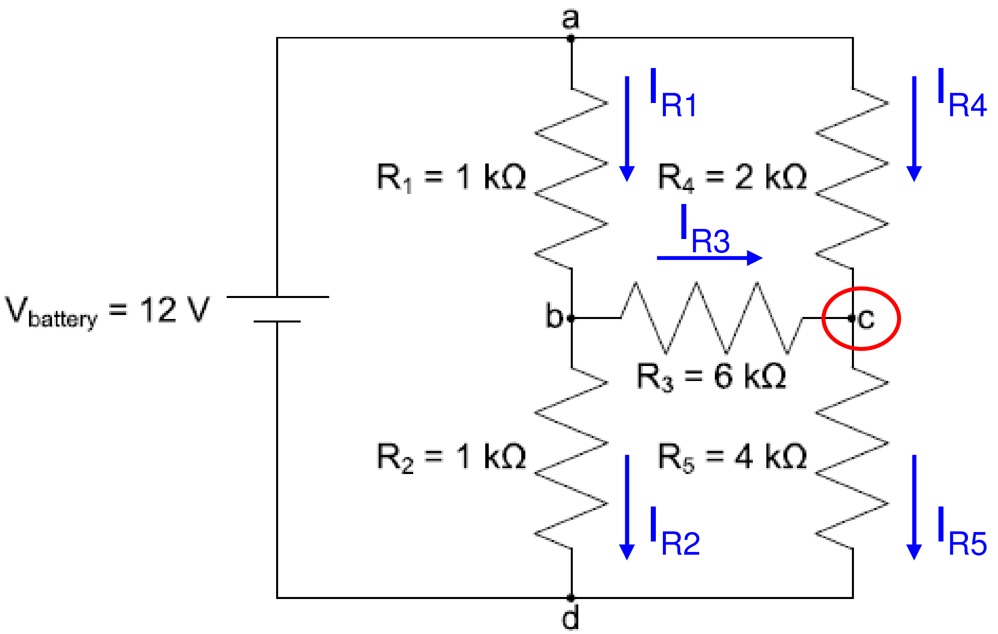
$$I_{R_2} = \frac{V_b - V_0}{R_2} = \frac{Vb - 0}{1000}$$

$$I_{R_3} = \frac{V_b - V_c}{R_3} = \frac{V_b - V_c}{6000}$$

$$I_{R_1} - I_{R_2} - I_{R_3} = 0 \rightarrow \frac{3}{250} - \frac{13}{6000} V_b + \frac{1}{6000} V_c = 0$$

$13V_b - V_c = 72$ (1)

Example - Applying KCL to a Circuit



KCL, node c

$I_{R5} = I_{R3} + I_{R4}$

Ohm's Law

$$I_{R_3} = \frac{V_b - V_c}{R_3} = \frac{Vb - V_c}{6000}$$

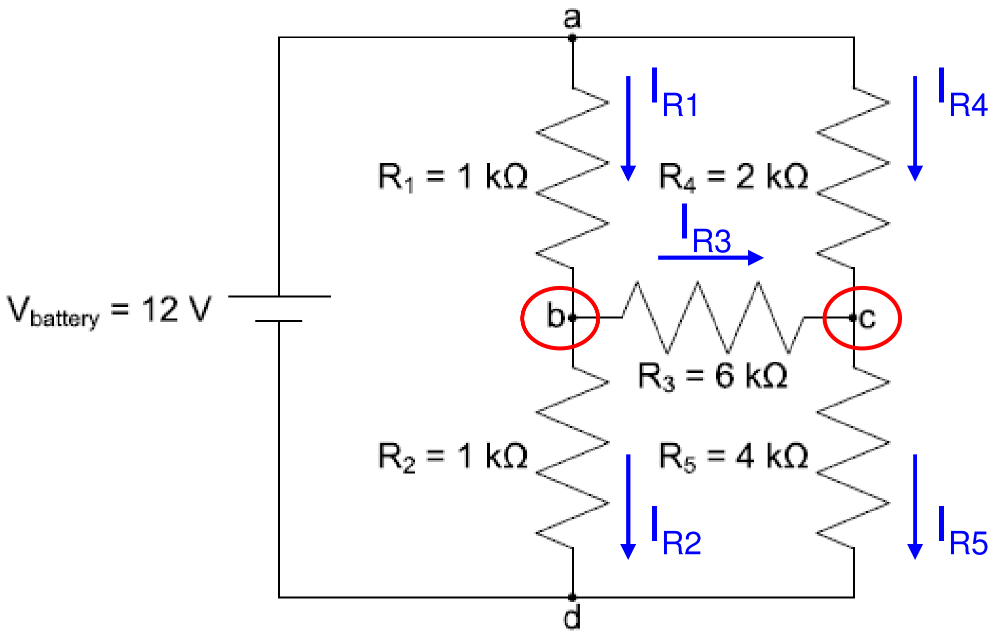
$$I_{R_4} = \frac{12 - V_c}{R_4} = \frac{12 - V_c}{2000}$$

$$I_{R_5} = \frac{V_c - V_d}{R_5} = \frac{V_c - 0}{4000}$$

$$I_{R_3} + I_{R_4} - I_{R_5} = 0 \rightarrow \frac{3}{500} - \frac{1}{6000} V_b + \frac{11}{12000} V_c = 0$$

$-2V_b + 11\ V_c = 72$ (2)

Example - Applying KCL to a Circuit



$13V_b - V_c = 72 \quad (1)$

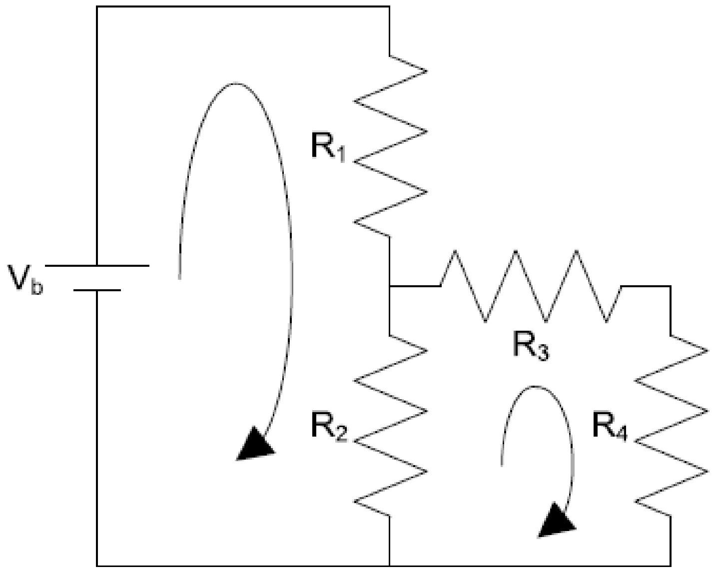
$-2V_b + 11\,V_c = 72 \quad (2)$

⇨ $V_b = 6.1277$ V, $V_c = 7.6596$ V ⇨ All currents

$$V_b < V_c \qquad I_{bc} < 0$$

Kirchoff's Voltage Law - KVL

Closed loop - a closed path which begins and ends in the same node.

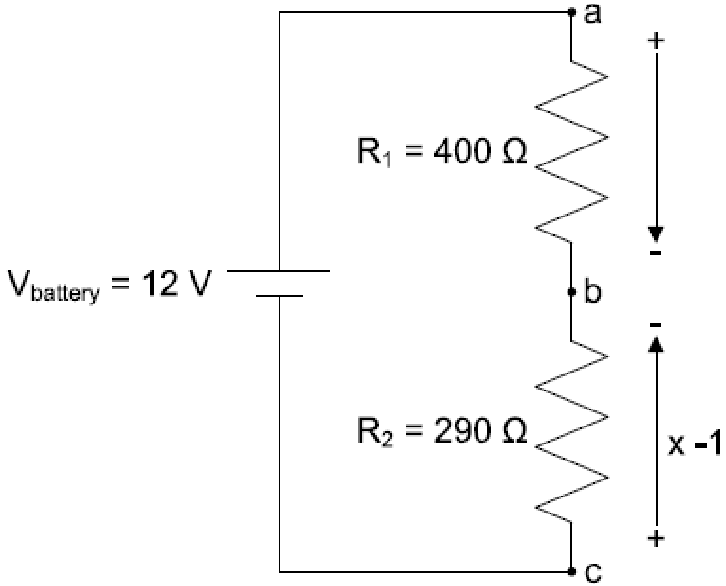


Kirchoff's Voltage Law - KVL

- KVL - The sum of the branch voltage drops around any closed loop is 0.
- Examples - Mountain excursion.

$$\sum V_{drops\ in\ a\ closed\ loop} = 0 \qquad (6.2)$$

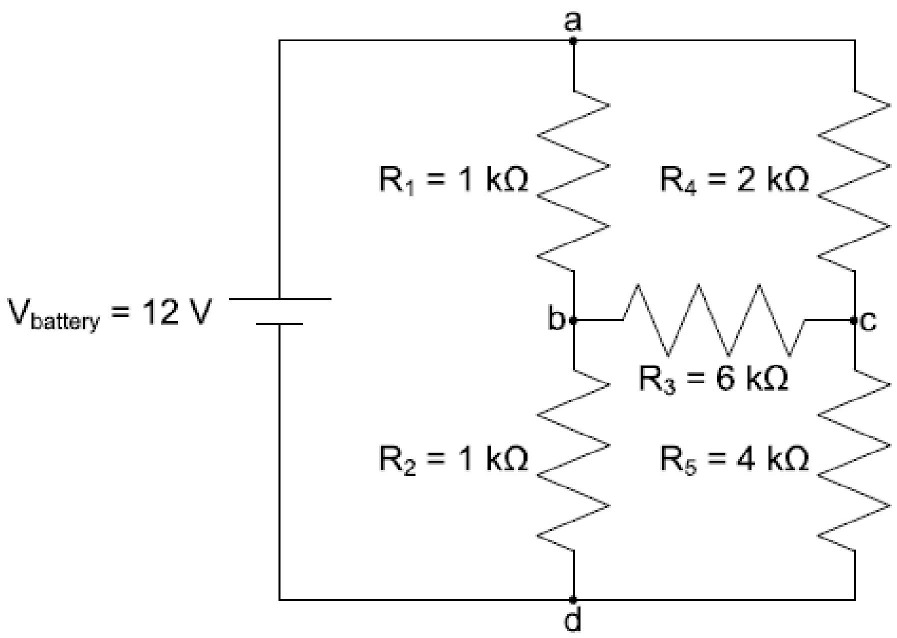
All drops should face the same direction.



Example - Applying KVL to a Circuit

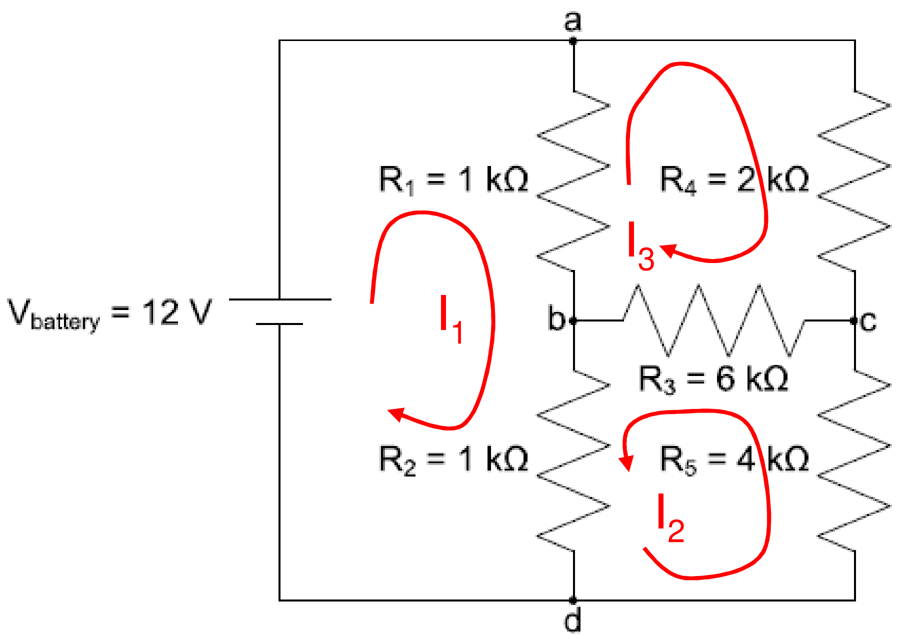
- Both KCL and KVL should give the same result to the same circuit.

How to find all the currents and voltages in the circuit below using KVL?

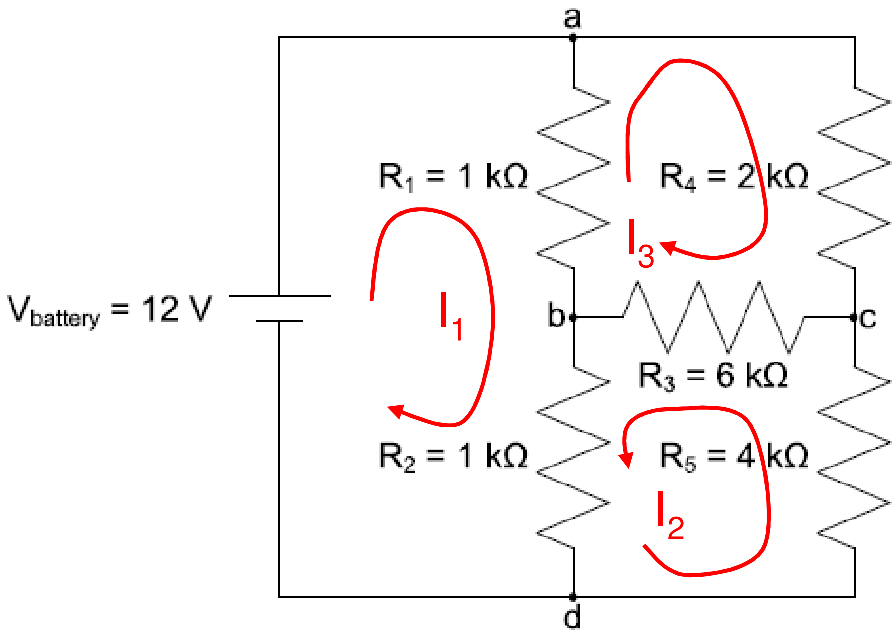


Example - Applying KVL to a Circuit

Mark the currents in loops.



Example - Applying KVL to a Circuit

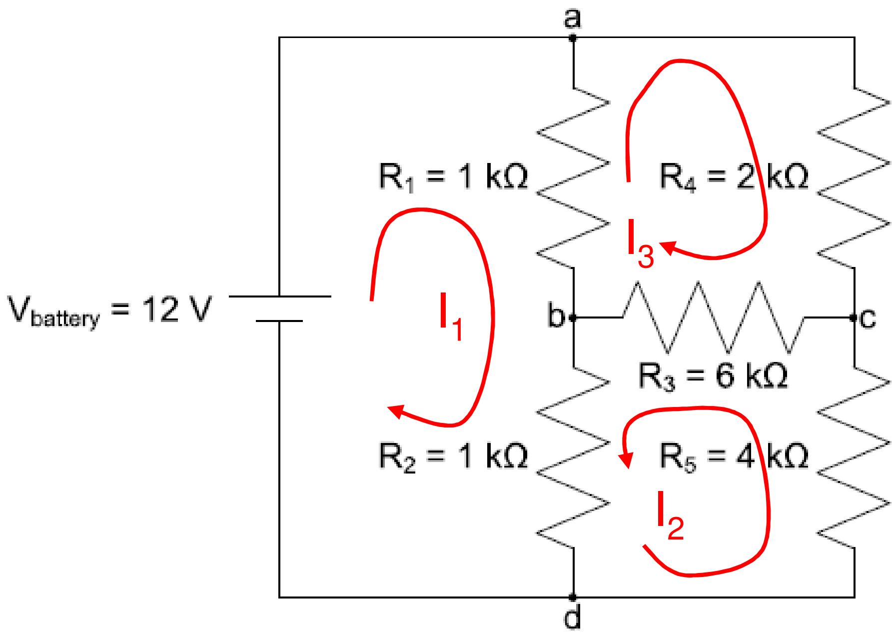


$$Loop\ \mathbf{I}_1 : a \rightarrow \mathbf{b} \rightarrow \mathbf{d} \rightarrow \mathbf{a}$$

$$R_1(I_1 - I_3) + R_2(I_1 + I_2) - V_{battery} = 0$$

$$1000(I_1 - I_3) + 1000(I_1 + I_2) - 12 = 0 \rightarrow 12 - 2000I_1 - 1000I_2 + 1000I_3 = 0$$

Example - Applying KVL to a Circuit

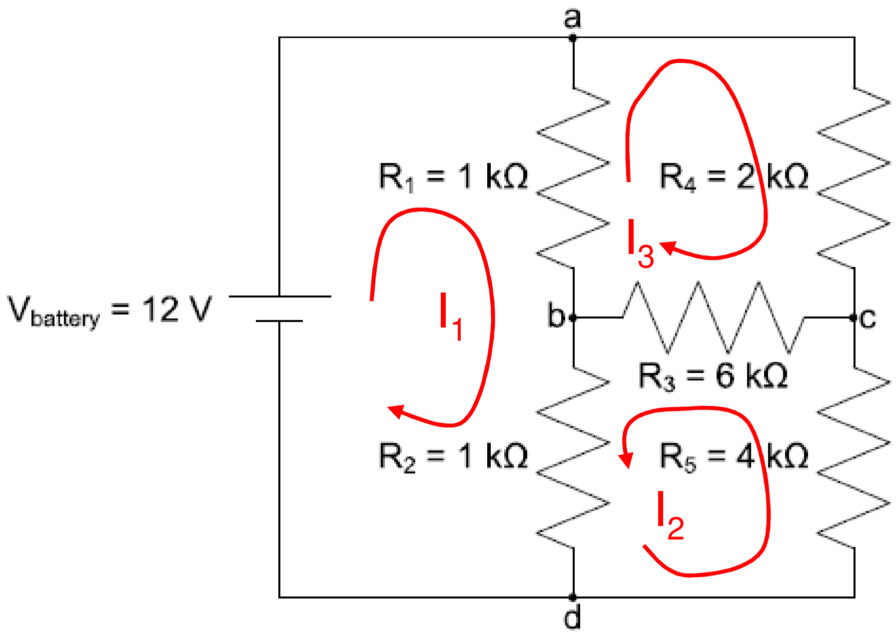


$$\textit{Loop}\ \mathrm{I}_3 : a \rightarrow \mathrm{c} \rightarrow \mathrm{b} \rightarrow \mathrm{a}$$

$$R_4 I_3 + R_3(I_2 + I_3) + R_1(I_3 - I_1) = 0$$

$$2000 I_3 + 6000(I_2 + I_3) + 1000(I_3 - I_1) = 0 \rightarrow -1000 I_1 + 6000 I_2 + 9000 I_3 = 0$$

Example - Applying KVL to a Circuit



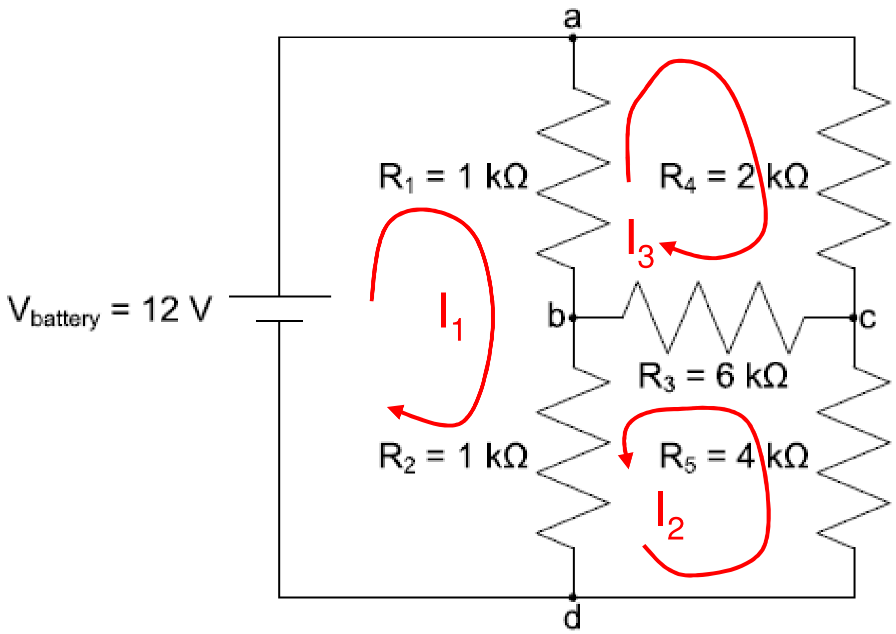
$$Loop\ \mathbf{I_2} : b \rightarrow \mathbf{d} \rightarrow \mathbf{c} \rightarrow \mathbf{b}$$

Mistake in the reader

$$R_5 I_2 + R_3(I_2 + I_3) + R_2(I_1 + I_2) = 0$$

$$4000I_2 + 6000(I_2 + I_3) + 1000(I_1 + I_2) = 0 \rightarrow 1000I_1 + 11000I_2 + 6000I_3 = 0$$

Example - Applying KVL to a Circuit

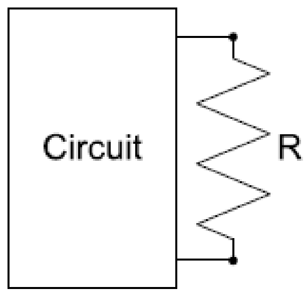


$I_1 = 0.008$ A, $I_2 = -0.0019$ A, $I_3 = 0.0022$A

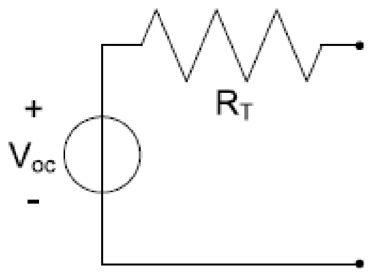
Note: the signs in the reader for I_2 and I_3 are wrong!!

Norton and Thevenin

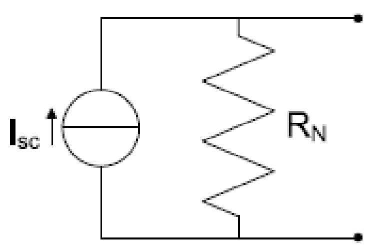
Simplify a complex circuit by a much simpler equivalent circuit using either (1) a voltage source with an equivalent resistor (Thevenin) (2) a current source with an equivalent resistor (Norton)



A complex circuit



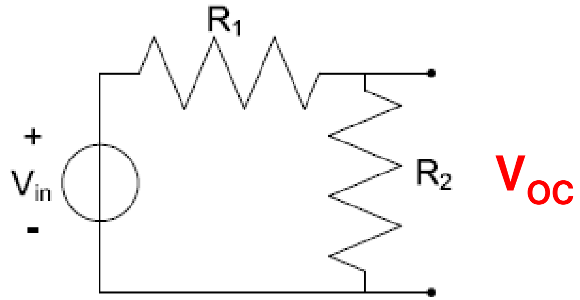
Thevenin



Norton

Norton and Thevenin

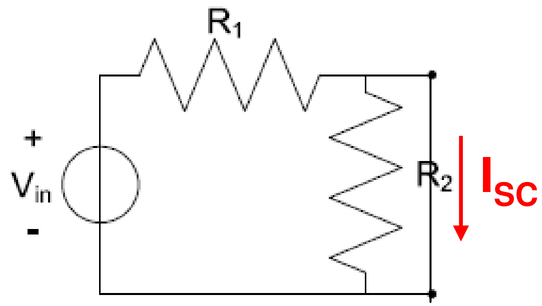
How to determine V_{OC}?



- Remove the load, leaving the load terminals open-circuited.
- Calculate the open-circuit voltage V_{OC}.

Norton and Thevenin

How to determine I_{SC}?



- Replace the load with a short circuit.
- Calculate the short circuit current I_{SC}.

Norton and Thevenin

$$R_T = R_N = \frac{V_{oc}}{I_{sc}}. \tag{7.1}$$

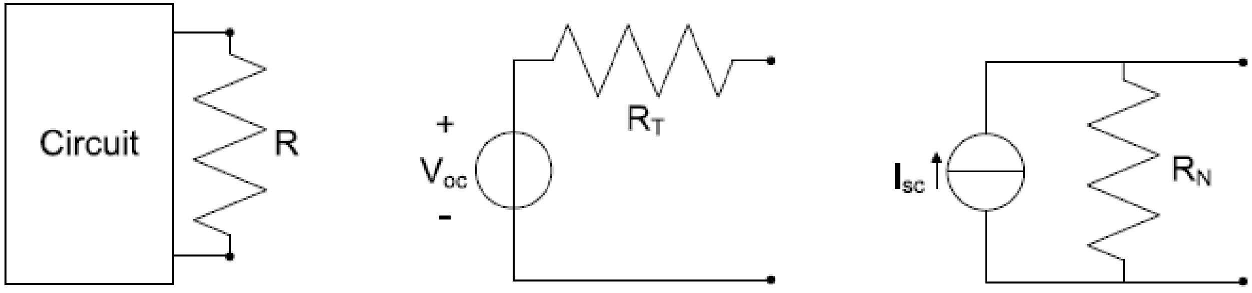
Thevenin and Norton resistances are equal;

The Thevinin voltage is equal to the Norton current times the Norton resistance;

The Norton current is equal to the Thevenin voltage divided by the Thevenin resistance.

Norton and Thevenin

Another way to determine R_T, R_N?



A complex circuit | Thevenin | Norton

$R_T = R_N$

- Remove the load
- Zero all independent voltage and current sources (e.g. short-circuit the voltage source and open-circuit the current source)
- Compute the total resistance between load terminals